Real-Time Uncertainty Estimation of Autonomous Guided Vehicles Trajectory taking into account Correlated and Uncorrelated Effects

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Abstract - This paper presents the description of a novel uncertainty estimation method employed for the navigation of Autonomous Guided Vehicles. In the proposed algorithm the uncertainty of the odometric navigation system is estimated as a function of the actual manoeuvre being carried out, which is identified by navigation data itself. The result is a recursive method for estimating the evolution of spatial uncertainty which takes into account unknown systematic effects and uncorrelated effects due to kinematic model uncertainty. The method is explained starting from the measurement models and its parameters as a function of the actual manoeuvres. A verification of covariance propagation estimate due to systematic effects was carried out by means of Monte Carlo simulation method. Experimental verification was carried out using an autonomous vehicle. Compatibility between a reference environment referred measurement systems work at a slower rate and need visible artificial external references to achieve a measurement with respect to the environment where the robot is moving [2]. These systems make use of augmented state and noise modelisation [3, 4, 5, 7, 14] must be used by means of fusing the information coming from sensors of the above categories. In this way it is guaranteed a high data rate and an uncertainty in pose estimation which is always bounded. In order to gain good accuracy from the fusion algorithm it is fundamental an accurate and real-time uncertainty estimation of both the incremental and the environment referred measurement systems. While for the environment referred system the uncertainty is a direct function of some known parameters (the number of visible targets, the vehicle speed, etc), for the incremental ones it is a function of some kinematics parameters, of kinematic model and of the state history (in particular the trajectory attitude). Furthermore the kinematics parameters uncertainty correlation must be considered. For the above reasons the uncertainty estimation cannot be based upon methods of combined standard uncertainty [13], but must be itself a recursive, state dependant function.

Many methods of spatial uncertainty estimation based upon Kalman filtering data fusion consider covariance propagation unaffected from parameters uncertainty correlation. This leads to uncertain regions (ellipses) that decrease as a function of the trajectory instead to grow as actually is the case (for trajectories basically in one direction, i.e. non-closed trajectories that actually lead to partially uncertain compensation). In order to overcome this problem some of them make use of augmented state and noise modelisation [3, 4, 5, 6, 7].

The covariance propagation method proposed in the present paper is based on a recursive covariance estimation that embodies the actual trajectory and takes into account both correlation of kinematic parameters and uncorrelated effects due to kinematic model uncertainty [16]. The correlation issue is achieved by adding an integral matrix that recourses together with the covariance one. The kinematic model uncertainty issue is achieved by introducing a stochastic term to the model which leads to an adding covariance term proportional to the pose increments. The propagation of uncertainty due to correlation (throughout a path) of parameters of robot non-linear kinematic model has been verified by means of Monte Carlo simulation, as outlined in [15]. Instead, the effects of
Kinematic model uncertainty cannot be verified with Monte Carlo simulation, they have been verified experimentally. Benefits of the proposed method are that the actual manoeuvres uncertainty and kinematic model uncertainty are taken into account when fusing odometers with inertial sensors. The proposed method allows to avoid the use of augmented state and noise modelisation thus simplifying and fastening the real-time implementation.

The algorithm was implemented on a differential drive autonomous vehicle. A PXI (National Instruments) with an embedded real-time operating system (RTOS) was used to control the robot and implement the on-line uncertainty estimation.

II. ON-LINE UNCERTAINTY ESTIMATION

The kinematics and navigation equations of the differential drive are explained in this section. An odometric system using two encoders on the driving wheels were installed on the vehicle.

The discrete form of the odometric navigation equations, neglecting the uncertainty factors, is the following:

\[
\begin{aligned}
x_{k+1} &= x_k + \pi \cdot \frac{n_{RL} \cdot R_L + n_{RR} \cdot R_R}{n_0} \cdot \cos(\delta_k) \\
y_{k+1} &= y_k + \pi \cdot \frac{n_{RL} \cdot R_L + n_{RR} \cdot R_R}{n_0} \cdot \sin(\delta_k) \\
\delta_{k+1} &= \delta_k + 2\pi \cdot \frac{n_{RL} \cdot R_L - n_{RR} \cdot R_R}{b}
\end{aligned}
\] (1)

The definition of symbols relating to the equations shown in the paper can be found in the list of symbols at the end.

In order to combine the incremental system measurements (mainly affected by drift) with an environment referred system, the uncertainty estimation at each step [4,6] must be known. Uncertainty is expressed in terms of the covariance matrix of the vector \(X_k = [x_k, y_k, \delta_k]\). In order to achieve the above covariance estimation, Equation 1 must be rewritten taking into account the pose increments noise:

\[
X_{k+1} = X_k + F_{wk} + [\Phi_{wk} \mid \xi_k]
\] (2)

where \(\Phi\) is a nonlinear function of \([n_{RL}, n_{RR}, n_0, R_L, R_R, b]\) that calculate the position and attitude increments at each iteration step. Parameter \(n_0\) is obviously constant because it is the number of pulses of the encoder for each revolution. Parameters \(n_{RL}\) and \(n_{RR}\) are neglected as a source of uncertainty because its errors are commonly compensated for between the present cycle and the ones that follow if the period \(T_c\) is small enough compared to the system dynamics. From the above considerations the vector \(w_k\) of variables that affect the accuracy of the estimation of the position and attitude as for the systematic uncertainty component, can be composed as follows:

\[
w_k = [R_L, R_R, b, \delta_k]
\] (3)

The vector \(\xi_k\) is a stochastic variable related to kinematic model uncertainty. It defines a random vector, its covariance matrix is diagonal with first two main elements equal, thus defining an uncertainty circle in a x-y space. It is used to describe the error that affects pose increment. This error is proportional to increment modulus, therefore the resulting covariance is proportional to the covariance of \(\xi_k\) by the square of increment modulus factor.

While the vector \(w_k\) refers to the uncertainty due to model variables’ correlation, the vector \(\xi_k\) refers to a stochastic component of the uncertainty due to kinematic model uncertainty.

Equation 2 can be used to evaluate the standard uncertainty of the estimation \(X_{k+1}\) if the standard uncertainty of the estimates \(X_k, w_k, \xi_k\) are known. Under the hypothesis that will be explained below the uncertainty can be expressed using the covariance matrix (refer to the list of symbols at the end of the paper for the definition of the various terms):

\[
C_{X_{k+1}} = C_{X_k} + \mathcal{J}_{X_k} \cdot C_{\xi_k} \cdot \mathcal{J}_{X_k}^T + \sum_{i=1}^2 \left( I_k \cdot S_{\xi_k} \cdot \mathcal{J}_{X_k} \right) + [\Phi_{wk} \mid \xi_k]
\] (4)

where the integral term \(I_k\) can be evaluated using the following synchronised recursion:

\[
I_k = I_{k-1} + \mathcal{J}_{X_{k-1}} \cdot S_{\xi_{k-1}}
\] (5)

- In Equation 4, on the left, there are five terms (one more term has been added with respect to the previous method in [16]). The first one is the covariance matrix of the preceding sample. The second term is the contribution of the parameters’ uncertainty as if they were uncorrelated between each other, i.e. \(E\{\varepsilon_{wk}(i) \cdot \varepsilon_{wk}(j)\} = 0 \forall i \neq j\), and as if the same parameter was also uncorrelated with itself over time (no systematic effect), i.e. \(E\{\varepsilon_{wk}(i) \cdot \varepsilon_{wk}(i)\} = 0 \forall k \neq h\). The standard uncertainties expressed in the covariance matrix \(C_{\xi_k}\) depend on the actual manoeuvre the robot is performing and are assumed to be uncorrelated thus leading to an orthogonal matrix. The third and fourth terms take into account full correlation of...
III. VERIFICATION OF COVARIANCE PROPAGATION

In this section are described two methods used for verification of the uncertainty propagation algorithm concerning systematic effects:

A. Deterministic Verification

The algorithm of uncertainty propagation due to systematic effects has been tested by means of simulations and experimental verification carried out for a set of representative trajectories: straight path, a curve of 90° and non-closed trajectories.

The calculation correctness has been tested by reconstructing a trajectory changed respect to the nominal one. The changed trajectories are obtained by changing each influence parameter one at a time, assigning to each parameter its variate, i.e. one of the two extreme values of its assigned coverage interval, and leaving the others parameters to their nominal value. Each trajectory is reconstructed from the same odometric data acquired during a real path. As expected, the simulations show that the changed trajectory is the envelope of the uncertainty ellipses calculated with the same coverage factor used to calculate the variate of the relative influence parameter (see Figure 2,3). The uncertainty ellipses represent a section of the multivariate normal

- a deterministic method was employed to test covariance matrix calculation correctness, it consists in making a comparison between a nominal and a changed trajectory;
- a Monte Carlo numeric simulation technique for the propagation of distributions was employed in order to verify that the model, which the propagation is applied to, under supposed conditions (distribution type and standard uncertainty of the input parameters known) is correct. In other words it was used to verify if at the end-point (but also in any other point) of a trajectory, the covariance estimated for model’s outputs is actually representative, under a specified level of confidence, of the robot’s attitude and position estimate. These two verification methods don’t concern the part of uncertainty propagation model regarding the uncorrelated effects of kinematic model uncertainty. It is important to notice that this last part can be separated from the others in the case of closed trajectories.

- the samples throughout the integration period, i.e. $E\{\varepsilon_w(i)\}_{\varepsilon_1}\{\varepsilon_w(i)\}_{\varepsilon_2} = 1 \forall k, \forall h$. In the case of closed trajectories the contributes of the second, third and fourth terms are such that at the end-point compensate between each other and the covariance matrix, without considering the contribute of the fifth term, returns to its initial value. This happens because these terms don’t take into account part of uncertainty that affect estimated increments and is due to the non-ideal kinematic model (model uncertainty). The fifth term is a diagonal matrix proportional to the modulus of the increments vector that takes account of uncertainty in estimation of the robot instantaneous centre of rotation. This will be better explained in §IV. In other words, the last three terms estimate the drift as a function of time. For long periods, these last three terms prevail. For short periods the first two terms prevail.

The hypothesis that there is complete correlation of each uncertainty factor $w_k(i)$ with itself over time but uncorrelation between the different factors needs to be explained for each factor separately:

- it is true for $R_L$ and $R_R$ if the effect of the load is considered constant throughout the path, while it is not true if the effect of the floor irregularities on the wheels’ radius is considered; moreover, load variations induces correlation effect between the two wheels’ radial dimensions and a temperature variation induces correlation between $R_L$, $R_R$ and $b$ as well;

the correlation holds for $\delta$ throughout the integration period, but it is not uncorrelated to the other uncertainty factors so that the uncertainties relate to this parameter are supposed to be underestimated. This has been verified by simulation results.

III. VERIFICATION OF COVARIANCE PROPAGATION ESTIMATE DUE TO SYSTEMATIC EFFECTS

In this section are described two methods used for verification of the uncertainty propagation algorithm concerning systematic effects:
To be published on IEEE Transactions on Instrumentation and Measurement June 2007

Fig. 4. Uncertainty propagation for a 90° curve.

Fig. 5. MC simulation for 90° curve. M= 10^5 iterations, 95% confidence ellipses.

Fig. 6. Uncertainty propagation for a straight path.

Fig. 7. MC simulation for straight path, 95% confidence. M=10^5

distribution and are calculate with a given confidence level (95%).

B. Monte Carlo Verification

A Monte Carlo simulation was carried out in order to: verify the hypothesis of gaussian distribution for the output end-position and attitude estimated; verify if the orientation of the estimated uncertainty ellipse and the entity of its principal components is reliable, in other words if the estimation of the covariance matrix is correct. To implement this technique we referred to [15] which outlines a ‘propagation of distributions’ approach to deriving the distribution of a measurand for any non-linear function and for any set of random inputs; the guide doesn’t still propose a method for multiple output systems like that used here. The Supplement’s approach assumes that the distributions of the random inputs are known exactly. We supposed that input parameters are all normally distributed and have a best estimate and a standard uncertainty which have been calculated by a calibration procedure.

Table I shows the parameters used for MC simulations:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best Estimate</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_x$</td>
<td>89.80 mm</td>
<td>$\sigma_{R_x} = 0.14$ mm</td>
</tr>
<tr>
<td>$R_y$</td>
<td>89.71 mm</td>
<td>$\sigma_{R_y} = 0.14$ mm</td>
</tr>
<tr>
<td>$b$</td>
<td>754.3 mm</td>
<td>$\sigma_b = 0.5$ mm</td>
</tr>
</tbody>
</table>

Note that attitude $\delta$ is not considered as an input parameter nor its covariance, because in the MC simulation it is an output value depending on the values of the other kinematic parameters indicated in Table I, thus its distribution will be an output of the simulation while in the propagation method its covariance is recursively calculated and associated to the influence parameters covariance matrix $C_{xy}$.

All the Monte Carlo simulations are referred to the end-position of trajectory. For each iteration the input parameters,
considered as influence parameters, have been all randomly drawn within their assumed normal distribution, each iteration carrying out an end-point \((x_i, y_i, \delta_i)\) reconstructed by the kinematic system model (see Equation 1). Choosing a sufficient number of iterations \(M\), the result of simulation is a samples set representative of the distribution of end-position (including attitude), its covariance matrix has been calculated and compared with that estimate by means of the recursive propagation method. The Equations 6 explain how the \(i\)-th normal variate parameter was calculated, \(\text{normrand} \) being the function for randomly choosing, for example, the \(R_R\) variate within its normal distribution with mean \(R_{R_i}\) (nominal value) and standard uncertainty \(\sigma_{R_i}\), giving the \(i\)-th value \(R_{R, i}\).

\[
\begin{align*}
R_{R,j} &= \text{normrand}(R_{R_i}, \sigma_{R_i}) \\
R_{b,j} &= \text{normrand}(R_{R_i}, \sigma_{R_i}) \\
b_{i} &= \text{normrand}(b_{0, i}, \sigma_{b}) \\
(x_i, y_i, \delta_i) &= f(R_{R,j}, R_{b,j}, n_{R}, n_{b})
\end{align*}
\]

For all the simulations, a value \(M = 10^5\) iterations have been chosen.

Concerning to the \((x, y)\) end-point position of tested trajectories, Monte Carlo simulations show a good correspondence both for the orientation of the covariance matrix eigenvectors and for its eigenvalues (standard deviations). It was also verified that, assuming a gaussian distribution for the influence parameters \(R_\alpha\) and \(R_\delta\) and \(b\), the distribution of the final pose estimated \((x, y, \delta)\) is Gaussian too. A chi-square test for all the samples-set analyzed gave a positive results for normal distribution hypothesis at 95% of confidence level.

Also, has been found that the hypothesis of attitude uncertainty underestimated, as explained in §II, is verified, in fact the mixed terms \(\sigma_{R, \delta}, \sigma_{\alpha, \delta}\) in the Monte Carlo covariance matrix are larger than the correspondent terms of the covariance matrix estimated with the propagation method here presented and discussed (Equations 7 are a numerical example of estimated covariance and Monte Carlo computed covariance related to the end-point of trajectory in Figure 4.

\[
C_{\text{est, MC}} = \begin{bmatrix}
-0.000886 & -0.000943 & -0.000875 \\
-0.000943 & -0.000795 & 0.000521 \\
-0.000875 & 0.000521 & 0.000977
\end{bmatrix}
\]

The statistic model used to simulate the thermal and load correlation effect is reported here.

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The mixed terms of third column are different between the two matrices (the matrices are symmetric). Instead, if we compare only the covariance submatrix related to \((x, y)\) end-point distribution and the term \(\sigma_{R, \delta}\) there is a good correspondence between the two matrices. In the case considered as example, while the difference between the 2-norm of diagonalized 2x2 \(C_{\text{est}}\) submatrices is 1%, the same difference between the complete 3x3 diagonalized \(C_{\text{est}, \delta}\) covariance matrices is 27%.

A Monte Carlo Simulation was used also for studying the effect of possible factors that can give rise to a correlation between the influence factors which has not been considered in the propagation method, such as varying temperature and varying load effect.

The load variation correlation effect was supposed to interest the parameters \(R_\alpha\) and \(R_\delta\) and has a behaviour similar to that of temperature, but in a slightly different principal direction of correlation. This effect could be taken into account by introducing in parameters’ covariance matrix an estimate of the correlation coefficient.

The statistic model used to simulate the thermal and load correlation effect is reported here.

**Thermal correlation model used**

If \(\Delta R_{R, \Delta T_{\max}}, \Delta R_{L, \Delta T_{\max}}, \Delta b_{\Delta T_{\max}}\) are the parameters’ variations caused by a thermal expansion because of a \(\Delta T_{\max}\) variation with respect to the nominal environment temperature and \(\Delta T\) is the random variable, drawn from a supposed rectangular distribution with a \([-\Delta T_{\max}, \Delta T_{\max}]\) interval and relative to iteration \(i\):
\[
\begin{align*}
R_{i+1} &= R_{i} + \Delta R_{i+1} \cdot \Delta T_i / \Delta T_{\text{max}} \\
R_{i+1} \cdot \Delta T_i &= R_{i} \cdot \Delta T_{\text{max}} \\
b_{i+1} &= b_{i} + \Delta b_{i} \cdot \Delta T_i / \Delta T_{\text{max}}
\end{align*}
\]

where the integral terms take into account the trajectory path the covariance matrix returns to its initial value. This is due to the effect of the recursive state dependent function where the integral terms takes into account the trajectory history and the systematic effects leads the covariance matrix to reduce itself to its initial value thus exploiting a compensation effect.

This is a well-suited result if the kinematic model is considered not affected by uncertainty. The last term of Equation 4 is a contribution due to a random error which is proportional to the amount of the increment itself. This random uncertainty can arise when the model is not fully representative of the true kinematic system because of wheels’ slippage, wheels’ misalignment and varying wheelbase cause a source of uncertainty in the instantaneous centre of rotation of the robot. The effect of this uncorrelated uncertainty source is proportional to the actual pose increment and therefore is assumed to increase the whole pose uncertainty in a way independent by the actual manoeuvre except by the distance covered by the robot wheels and by attitude variations during integration time.

The estimate of model uncertainty cannot be performed by means of simulation, a calibration procedure has to be carried out by means of the experimental vehicle in order to define the three elements of the covariance matrix of vector \(\xi\). A good test trajectory is a straight path forth and back to initial position in order to reduce the systematic effects contribution to uncertainty estimation.

A straight eight meters path has been tested both in forward and in backward direction, i.e. starting and arriving at the same position, in order to compare the uncertainty estimated in both the cases of taking into account or not the contribution of the term of kinematic model uncertainty (Figure 10).

In the previous section we verified the propagation of spatial uncertainty concerning the effect of kinematic parameters uncertainty (vector \(w\)). As confirmed by simulation results, the uncertainty propagation algorithm for these parameters is such that for a strait forward-backward path the covariance matrix returns to its initial value. This is due to the effect of the recursive state dependent function where the integral terms takes into account the trajectory history and the systematic effects leads the covariance matrix to reduce itself to its initial value thus exploiting a compensation effect.

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In the first case, when the robot returns to its starting position, the covariance matrix returns to its null initial value because of the systematic correlation effect of the kinematic parameters; in the second case, when moving backward, the uncertainty ellipses start decreasing by the systematic correlation effect but at a certain point they restart to increase by the prevailing effect of model uncertainty errors taken into account.

A set of forward-backward straight paths of different lengths (2,4,6,8 meters) was repeated with a nominal load and a constant velocity of 0.3 m/s in three different initial attitudes in such a way that the robot met different floor irregularities. Figure 11 shows the end-point estimation differences between the odometric system and the reference system and the estimated uncertainty ellipses computed with a 95% confidence level. The calibration procedure has led to obtain compatibility with the uncertainty ellipses in 95% of the trials. In order to verify the distribution of end-point measurements, covariance matrix of the experimental data has been computed showing good correspondence with the covariance matrix computed with the propagation method.
V. EXPERIMENTAL RESULTS

In order to verify the proposed technique experimentally, a mock-up of differential drive robot 1.1 m diameter (see Figure 1) was used. Two incremental encoders were mounted on the vehicle.

The encoders have 1000 ppr. The control references were generated and navigation recursion were computed by an industrial PXI embedded system with an analogue board and a digital one.

In order to verify the odometric uncertainty propagation algorithm, a triangulation sensor based upon an optical scanner mounted on the robot and infrared transmitters fixed in the environment [2,11] were used. The angular uncertainty is ±47 arc-seconds, and the position accuracy in the centre of a 5m square room is ±1 mm (confidence level ±2σ). Figure 12 shows the uncertainty ellipses caused by the systematic and uncorrelated effects discussed in § II as a function of the path (the ellipses are generated taking into account the first two rows and the first two columns of the matrix C Xk, see Equation 4).

To show the effectiveness of the proposed algorithm in evaluating the uncertainty ellipses, a set of 4 paths was repeated changing the vehicle load and velocity: on the first path the vehicle had a nominal load and a velocity equal to 0.3 m/s; on the second the vehicle had the nominal load plus 10 kg and a velocity equal to 0.3 m/s; on the third the vehicle had a nominal load and a velocity equal to 0.6 m/s; on the fourth the vehicle had the nominal load plus 10 kg and a velocity equal to 0.6 m/s. For each set, two initial attitude (see θ in Figure 12) and two different position of the load have been tested. Figure 13 shows the end-point estimation differences between the proposed algorithm and the reference system and the estimated uncertainty ellipse computed with a 95% confidence level.

As seen in the Monte Carlo simulations, the correlation increment gives in general a thin ellipse (i.e. a preferred direction) and this is caused by the fact that the influence parameters give rise to the same direction in end-point estimation errors. The experimental results show that the uncorrelated effect on the pose estimation uncertainty is to enlarge the ellipses in both their main directions.

There is a huge variation in the geometry and orientation of the ellipses as a function of path. This clearly shows the need for the covariance estimation of uncertainty and not only the standard uncertainties σx and σy, which could lead to inaccurate fusion with an environment referred system (for example with laser scanner position estimation).

VI. CONCLUSIONS

This paper presents a recursive method for the estimation of the evolution of spatial uncertainty was developed taking into account the systematic and uncorrelated effects as well.
Calibration of the uncertainty parameters was carried out as a function of the different manoeuvres.

The uncertainty ellipses were thus computed as a function of the path. The method’s calculations, as for systematic effects, were tested by means of Monte Carlo simulations employed in order to verify the propagation of the input parameters’ distributions and the effect of thermal and load correlations not taken into account. A calibration of the model uncertainty that gives rise to uncorrelated effect was carried out experimentally.

The estimated errors in the end point measurement was experimentally carried out using a reference system with different loads and speed conditions. The compatibility with the uncertainty ellipses was verified 95% of the trials.

LIST OF SYMBOLS

\( (x,y,\delta) \) the sensor fusion estimated position and attitude with respect to the fixed reference of the reference point \( P_0 \) on the vehicle

\( X_k \) \((x,y,\delta)\) position and attitude vector at step \( k \)

\( C_V \) covariance matrix of the vector \( V \)

\( R_n, R_l \) the right and left driver wheels’ radius

\( n_R, n_L \) the number of counts from the driving right and left encoder

\( n_o \) the num. of counts of the driving encoder in one turn

\( b \) the wheel base of the differential drive robot

\( \sigma_\lambda \) the standard uncertainty in parameter \( \lambda \)

\( \epsilon_\lambda \) the uncertainty in parameter \( \lambda \) defined with an assigned coverage factor

\( \Phi_{bs} \) Jacobian matrix of the non linear function \( \Phi_b \)

\( C_{aa} \) diagonal matrix with covariances of parameters of \( w_\lambda \) vector

\( S_{aa} \) matrix whose elements are square root of \( C_{aa} \) elements

\( C_{\hat{w}} \) diagonal matrix estimating uncorrelated model uncertainty with covariances of pose increments

\( e_\xi \) random error vector affecting pose increments

MC Monte Carlo

REFERENCES


