Sensor Fusion of Inertial-Odometric Navigation as a Function of the Actual Manoeuvres of Autonomous Guided Vehicles

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Abstract: This paper presents a description of the ‘sensor fusion’ algorithm for a navigation system suited for Autonomous Guided Vehicles that use two navigation systems: an odometric one which uses encoders mounted on the vehicle wheels and an inertial one which uses a gyroscope. In the proposed algorithm the accuracy of both navigation systems is estimated as a function of the actual manoeuvre being carried out, which is identified by navigation data, and then the output of both sensors are fused taking into account the accuracy ratios. The method is explained starting from the measurement models and its calibration as a function of the different manoeuvres. Experimental calibration of the proposed algorithm was carried out using a scaled mock-up of an industrial robot. While the mean absolute value of the estimated error in distance over the path is 49 mm when using odometric navigation and 28 mm when using odometric plus inertial attitude navigation, it is only 15 mm when carrying out data fusion between the two measurement systems. A recursive method for estimating the evolution of spatial uncertainty, which also takes into account the systematic effects, is proposed.

1. Introduction

The use of Autonomous Guided Vehicles is widespread in a huge number of fields such as factories, ports, hospitals, farms, etc., yet measuring a vehicle’s attitude and position is still a challenging problem. The measurement systems currently used may be divided into direct, relative and absolute guidance. Though the first category is made up of systems which are the most reliable, such as wire-guidance, magnet-guidance, etc., they are also systems which suffer from the considerable problem of path planning. If the path has to be changed, a certain number of hours are required to install the cable inside the floor and the guidance system must be stopped during installation. The relative or dead-reckoning methods, such as encoders, gyroscopes, ultrasound, etc., have the considerable advantage of being totally self-contained inside the robot, relatively simple to use and able to guarantee a high data rate. On the other hand, since these systems integrate relative increments, errors grow considerably over time [1,2,4,5,6,7,8,9]. Absolute guidance makes use of external references to achieve an absolute measurement with respect to the environment where the robot is moving [3]. These systems are more complicated than the relative ones, work at a slower rate, and lead to the problem of the visibility of the targets needed during the robot’s path. However, since they measure the robot’s position and attitude with respect to absolute references (targets), the error is always bounded and absolute repeatability guaranteed [3,4,5,6,7,8,9].

From the above considerations it is clear why many systems currently make use of both a relative and an absolute system. In this configuration the major challenge is to reduce the number of absolute targets as much as possible by increasing the accuracy of the relative system [10,12] which is made up of the transducers and the recursive algorithm. In this way integrated relative-absolute navigation systems can be implemented in factories where the installation and optimal visibility of the targets is not possible throughout the entire path, e.g. partially external path, or where there are long corridors, narrow passages, obstacles in the plane of laser scanning, or simply where the configuration of the factory does not make it possible to achieve the needed accuracy.

In [10,11,13] the data from encoders and gyroscope was combined using a Kalman filter. Generally in this case the sensors’ attitude estimates are combined taking into account a fixed uncertainty that had previously been calibrated. In [11,12] the important problem of navigation loop integrity is addressed from a data fusion point of view. In particular
[11] gives a criteria to fuse data between an absolute measurement system (GPS) and an inertial relative one (IMU) in order to minimise the influence of sensors faults. In [12], based on a study of the physical interaction between the ground and the vehicle, the encoder and gyroscope data were combined taking into account the actual data when their outputs differ significantly from each other. This technique helps reduce the significant errors that can occur due to floor irregularities. In [13] it is pointed out that the characteristics of the floor upon which the mobile robot navigates can be taken into account in the form of vehicle motion constraints in order to improve the accuracy of the position and attitude estimation algorithm that makes use of data fusion between odometers, inertial units and GPS units.

In the method presented in this paper, encoder and gyroscope data are combined taking into account their uncertainty as a function of the actual manoeuvre the robot is performing. This is accomplished by using a statistical filter that is active throughout the whole path. To achieve this goal both a data fusion algorithm that considers the actual manoeuvre the robot is performing and a procedure to calibrate the sensors off-line for the different manoeuvres are proposed. The problem of adaptively estimating the unknown noise statistics from data is presently of great interest [8]; in this case, a technique which takes into account the actual sensor uncertainties using a fuzzy logic method applied to the data fusion between an absolute and a relative measurement system is discussed.

In order to fuse encoder data with gyroscope data, only the uncertainty of the attitude increments has to be estimated. To fuse the data of the encoder-gyroscope estimation of position and attitude with an absolute system, the evolution of the uncertainty as a function of time must also be estimated. For this purpose a recursive algorithm for uncertainty estimation is developed in the present work which makes it possible to calculate the corresponding uncertainty ellipse for each position estimation [14]. For this purpose it was fundamental to take into account the correlation between the uncertainty parameters.

The algorithm proposed was implemented on a mock-up of a scaled industrial robot equipped with driver-steering motors, power and logical drivers. A PXI system having an industrial PC with a real-time operating system embedded in it was installed onboard the robot. The industrial PC was connected via an internal bus to an analogue board and a digital board. The whole system can therefore be used to control the robot’s trajectory and to estimate its position and attitude in real-time. This mock-up was used to estimate experimentally the accuracy of the proposed sensor-fusion algorithm.

The paper is organised as follows: after this introduction the equations at the basis of data fusion algorithm are discussed and developed in §2. A complete procedure for the calibration of the data fusion algorithm is detailed in §3. The algorithm is verified experimentally in §4 by means of a scaled mock-up of an industrial robot. An appendix at the end is useful for deeper comprehension of equations in §2.

2. Data-fusion algorithm and uncertainty evaluation

The kinematics and navigation equations of the three wheeled vehicle with one driving-steering wheel and two fixed wheels in axis which was developed for this study are explained in this section. Two navigation systems were installed on the vehicle:

1. an odometric system using two encoders on the driving-steering wheel: one to measure the steering angle, the other to measure the angular increments;
2. an inertial system using a gyroscope to estimate the vehicle’s attitude.

The definition of symbols relating to the equations shown in the paper can be found in the list of symbols at the end. The discrete form of the Inertial-Odometric navigation equations is the following:

\[
\begin{align*}
\delta_{x+1}^E &= \delta_x + \frac{2\pi}{n_0} \cdot n_k \cdot R \cdot \cos(\alpha_k + \alpha_0) \cos(\delta_x) \\
\delta_{y+1}^E &= \delta_y + \frac{2\pi}{n_0} \cdot n_k \cdot R \cdot \cos(\alpha_k + \alpha_0) \sin(\delta_x) \\
\delta_{\phi+1}^E &= \delta_\phi + \frac{2\pi}{n_0} \cdot n_k \cdot R \cdot \sin(\alpha_k + \alpha_0) \frac{1}{b} \\
\delta_{\phi+1}^G &= \delta_\phi + \varphi_c \cdot G(\delta_\phi^G)
\end{align*}
\]
The output of equations 1 and 2 are combined by the data-fusion algorithm in order to estimate the best guess of the two attitude increments. The procedure is as follows: first the two navigation algorithm increments (eq. 3) are estimated, then the type of manoeuvre the vehicle is actually undergoing is estimated using the inertial-odometric data (Tab. 2), then the accuracy ratio of the output of the two navigation systems is estimated using table 1, then the data-fusion algorithm combines the two increments (eq. 6), and finally the output of the data-fusion algorithm is added to \( \delta_k \) to achieve \( \delta_{k+1} \) (eq. 3).

To formalise the data-fusion algorithm it is necessary to consider the increments:

\[
\begin{align*}
\Delta x_k^E &= x_{k+1}^E - x_k \\
\Delta y_k^E &= y_{k+1}^E - y_k \\
\Delta \delta_k^E &= \delta_{k+1}^E - \delta_k \\
\Delta \delta_k^G &= \delta_{k+1}^G - \delta_k
\end{align*}
\]  

(3)

as well as the accuracy of the attitude increments defined as: \( \pm \epsilon_{\Delta \delta}^\eta \), where \( \eta \) can be \( E \) or \( G \). The increments lead to the following equations:

\[
\begin{align*}
X_{k+1} &= X_k + \Delta x_k^E \\
Y_{k+1} &= Y_k + \Delta y_k^E \\
\delta_{k+1} &= \delta_k + DF(\Delta \delta_k^E, \Delta \delta_k^G)
\end{align*}
\]

(4)

Assuming Gaussian distribution of the uncertainties \( \sigma_{\Delta \delta}^E \) and \( \sigma_{\Delta \delta}^G \), the best attitude increment estimate that takes into account the attitude increments of both encoder and gyro is written in equation 5. In other words the data fusion algorithm of equation 4 is as follows:

\[
DF(\Delta \delta_k^E, \Delta \delta_k^G) = \frac{\left( \sigma_{\Delta \delta}^E \right)^2 + \left( \sigma_{\Delta \delta}^G \right)^2}{\left( \sigma_{\Delta \delta}^E \right)^2} = \frac{1}{\xi_R + 1}
\]

(5)

Actually, in order to fuse the data, only the ratio of the two uncertainties \( \sigma_{\Delta \delta}^E / \sigma_{\Delta \delta}^G \), defined as \( \xi_R \), is needed:

\[
\Delta \delta_k = \frac{\Delta \delta_k^E}{\xi_R + 1} + \frac{\Delta \delta_k^G}{\xi_R + 1}
\]

(6)

In order to combine the relative system measurements (mainly affected by drift) with the absolute system, the uncertainty estimation at each step \([5,7,9]\) must be known. Uncertainty is expressed in terms of the covariance matrix of the vector \( X_k = [x_k, y_k, \delta_k] \). In order to achieve the above covariance estimation, equation 4 can be rewritten as:

\[
X_{k+1} = X_k + \Phi(t_k) w_k
\]

(7)

where \( \Phi \) is a nonlinear function of \( [n_k, n_0, R, b, \alpha_k + \alpha_0, G(V_k), \delta_k] \) of which \( n_0 \) and \( b \) are considered constant. Parameter \( n_0 \) is obviously constant because it is the number of pulses of the encoder for each revolution. Parameter \( b \) is considered constant because in the common applications of mobile robots rigid structures are used which can in theory vary the distance between the wheel axes, but in practice the effect is less than one order of magnitude and can therefore be neglected with respect to the other effects. Parameter \( n_k \) is neglected as a source of uncertainty because its errors are commonly compensated for between the present cycle and the ones that follow if the period \( T_c \) is small enough compared to the system dynamics. From the above considerations the vector \( w_k \) of variables that affect the accuracy of the estimation of the position and attitude can be composed as follows:

\[
w_k = [R, \alpha_k + \alpha_0, G(V_k), \delta_k]
\]

(8)

Equation 7 can be used to evaluate the standard uncertainty of the estimation \( X_{k+1} \) if the standard uncertainty of the estimates \( X_k \) and \( w_k \) are known. The uncertainty can be expressed using the covariance matrix (refer to the Appendix for the definition of the various terms):

\[
C_{X_{k+1}} = C_{X_k} + \mathcal{C}_{\Phi_k} \cdot C_{w_k} \cdot \mathcal{C}_{\Phi_k}^T + \mathcal{C}_{\Phi_k} \cdot S_{\Phi_k} \cdot I_{\Phi_k} + I_k \cdot S_{w_k} \cdot \mathcal{C}_{\Phi_k}^T
\]

(9)
where the term $I_k$ can be evaluated using the following recursion:

$$I_k = I_{k-1} + S_{q_k} \cdot S_{u_k}$$  \hspace{1cm} (10)$$

In equation 9 there are four terms. The first one is the covariance matrix of the preceding sample. The second term is the contribution of the parameters’ uncertainty as if they were uncorrelated between each other, i.e.
\[E\{\epsilon_{wk}(i) \cdot \epsilon_{wk}(j)\} = 0 \quad \forall \; i \neq j,\]
and as if the same parameter was also uncorrelated to itself over time (no systematic effect), i.e.
\[E\{\epsilon_{wk}(i) \cdot \epsilon_{wh}(i)\} = 0 \quad \forall \; k \neq h.\]
The standard uncertainties expressed in the covariance matrix depend on the actual manoeuvre the robot is performing and are assumed to be uncorrelated thus leading to an orthogonal matrix. The third and fourth terms take into account full correlation of the samples throughout the integration period, i.e.
\[E\{\epsilon_{wk}(i) \cdot \epsilon_{wh}(i)\}/(\sigma_{wk}(i) \cdot \sigma_{wh}(i)) = 1 \quad \forall \; k, \forall \; h.\]
In other words, these last two terms estimate the drift as a function of time. For long periods, these last two terms prevail. For short periods the first two terms prevail.

The hypothesis that there is complete correlation of each uncertainty factor $w_k(i)$ with itself over time but uncorrelation between the different factors needs to be explained for each factor separately:

- it is true for $R$ if the effect of the load is considered constant throughout the path, while it is not true if the effect of the floor irregularities on the wheel radius is considered;
- total correlation can be considered true for $\alpha_k + \alpha_0$ only when steering in only one direction, while steering in one direction and then in the other can lead to compensation so that total correlation means overestimating the angle uncertainty ;
- it can be considered true for the systematic component of $G(V_k)$ if it is considered constant during the integration period; the random part is therefore neglected;
- the correlation holds for $\delta_k$ throughout the integration period, but it is not uncorrelated to the other uncertainty factors so that the uncertainty due to this parameter is underestimated.

A procedure for estimating $C_{wk}$, and thus uncertainty, as a function of time and manoeuvre history is proposed in the following section.

3. Uncertainty calibration procedure and results

In order to estimate uncertainty, the covariance matrix $C_{wk}$ has to be characterised as a function of the different manoeuvres the vehicle can make during travel. Four kinds of manoeuvres were defined for the purposes of this study: standing, straight path, curving, steering (refer to table 1). It is important to note that in order to minimise the systematic effects, it was assumed that a complete calibration of the measurement model had been performed [15] before starting the following procedure.

The elements of $C_{wk}$ were estimated at each step following this procedure.

1. $\sigma_{\delta_k}$ was estimated using the preceding sample (at the starting position the initial value must be known).
2. $\sigma_{\alpha_k}$ was estimated during a straight path and assumed to be constant as a function of the manoeuvre. This assumption was made because during steering or curving pneumatic deformation is mostly torsional thus leading to variations in the $\alpha$ parameters. Variations in R can occur but are considered negligible. In general, the wheel radius can vary according to load, floor conditions and wear. Wear leads to a systematic effect that leads to the need for on-line calibration [15] and thus compensation.
3. $\sigma_{\delta_k + \alpha_0}$ was estimated as a function of the manoeuvres. This value together with the one estimated in step 2 was used to calculate $\sigma_{\delta_k}^E$.
4. $\sigma_{G(V_k)}$ was estimated as a function of the manoeuvres. It was then used to estimate $\sigma_{\delta_k}^G$.

3.1 Standing

During standing the relative accuracy of the gyroscope is very low due to the drift of its bias; this is not the case for the encoders. During standing the gyroscope output can vary from its ideal zero value due to the drift effect. The gyroscope increments were therefore not considered when the encoder driver wheel increments were zero. The uncertainty ratio $\xi_{\delta_k}$ was zero during standing.

3.2 Straight path

During a straight path or a path with the steering angle close to zero, vibrations affect the gyroscope’s accuracy and its output is comparable to its bias.
In order to evaluate the standard uncertainty of $\sigma_R$, the robot travelled a 5 meter long path 20 times varying the load within the range of possible values and maintaining the steering aligned to the straight heading direction. The initial position $x_I$ and the final position $x_F$ were measured using the triangulating measurement system described in [3]. The integrating steps were defined as $k = 1 \ldots N$ and $\sigma_R$ was estimated as follows:

$$\sigma_R = \left( \frac{2\pi}{n_0} \cdot \frac{N}{b} \cdot \sum_{k=1}^{N} n_k \right)^{-1} \cdot \sigma_{x_F-x_l}$$

(11)

where $\sigma_{x_F-x_l}$ is the standard deviation of the differences between the reference triangulating measurement system estimates and the odometric estimation.

In order to evaluate the standard uncertainty of $\alpha_{k+0}$ and $\sigma_G(\varphi_k)$, the robot travelled a 5 meter long path 10 times keeping the load constant and maintaining the steering aligned to the straight heading direction. The vehicle’s attitude at the starting $\delta_I$ and final $\delta_F$ positions was evaluated using the triangulating measurement system described in [3]. The vehicle’s attitude was also evaluated using the odometric and inertial systems both at the starting and the final positions. By computing the differences with respect to the laser reference system, the standard deviations $\sigma_{x_F-x_l}^{E}$ and $\sigma_{x_F-x_l}^{G}$ were computed. Taking into account the third equation of eq. 1, it was then possible to estimate $\alpha_{k+0}$ and $\sigma_G(\varphi_k)$:

$$\sigma_{\alpha_{k+0}} = \left( \frac{2\pi}{n_0} \cdot \frac{R}{b} \cdot \sum_{k=1}^{N} n_k \right)^{-1} \cdot \sigma_{\delta_F-\delta_l}^{E}$$

(12)

$$\sigma_G(\varphi_k) = (N \cdot T_C)^{-1} \cdot \sigma_{\delta_F-\delta_l}^{G}$$

(13)

and by means of $\alpha_{k+0}$ it was possible to estimate $\sigma_{\alpha_{k+0}}^{E}$ in straight path conditions:

$$\left( \sigma_{\alpha_{k+0}}^{E} \right) = \frac{2\pi}{n_0} \cdot \frac{R}{b} \cdot n_k \cdot \sigma_{\alpha_{k+0}}$$

(14)

and by means of $\sigma_G(\varphi_k)$ it was possible to estimate $\sigma_{\alpha_{k+0}}^{G}$ in straight path conditions:

$$\left( \sigma_{\alpha_{k+0}}^{G} \right) = T_C \cdot \sigma_G(\varphi_k)$$

(15)

It is worth noting that estimating $\left( \sigma_{\alpha_{k+0}}^{G} \right)$ as shown in eq. 15 means ignoring the effect of velocity, which can be non negligible. In the present study this simplification was made for two reasons: the first is that, for the most common industrial applications, during straight path traveling the velocity is constant and has a predetermined known value; the second is the need to simplify. However, for some particular applications it could be necessary to estimate this uncertainty as a function of velocity.

During straight path travel, the uncertainties that were estimated on the mock-up of the industrial robot described in the following paragraph were: $\sigma_R = 0.1 \cdot 10^{-3}$ m; $\alpha_{k+0} = 0.0013$ rad; $\sigma_G = 0.0025$ rad/s (note that the calculations were evaluated at the nominal velocity). The ratio of the standard uncertainties (eqs. 14 and 15) was computed to estimate the uncertainty ratio $\frac{\sigma_R}{\sigma_G}$ roughly equal to 0.25. From practical considerations this ratio could be considered constant even when changing velocity or the steering manoeuvre.

To underline the previous conclusions with an example, one of the gyroscope output sequences during straight path was taken and its FFT was computed (fig. 3). From the spectral analysis it is possible to see the gyroscope
sensitivity to vibrations induced by floor irregularities and also note that the mean value is different from zero. The attitude increment between the starting and final positions was 0.18° for the gyroscope and 0.05° for the reference system.

3.3 Curving

In order to evaluate the standard uncertainty of \( \sigma_{k+0} \) and \( \sigma_{G(\nu_k)} \) during curving, 10 circular paths were performed with the steering angle at 0.8 radiant and 10 with the steering angle at -0.8 radiant. It is important to note that the variations as a function of the steering angle were not taken into account, but simplified considering only one value of the steering angle. The load was kept constant and thus the wheel radius as well. At the end of each circular path, about three complete revolutions had been achieved. The vehicle’s attitude was evaluated using the triangulating measurement system described in [3] both at the starting and final positions. The vehicle’s attitude was also evaluated using the odometric and inertial systems. By computing the differences with respect to the laser reference system, the standard deviations \( \sigma_{k+0} \) and \( \sigma_{G(\nu_k)} \) were computed. Taking into account the third equation of eq. 1 it was then possible to estimate \( \sigma_{k+0} \) and \( \sigma_{G(\nu_k)} \):

\[
\sigma_{k+0} = \left( \frac{2\pi R}{n_0} \sum_{k=1}^{N} (n_k \cdot \cos(\alpha_k + \alpha_0)) \right)^{-1} \cdot \sigma_{G(\nu_k)}
\]

(16)

and by means of \( \sigma_{k+0} \) and \( \sigma_R \) it was possible to estimate \( \sigma_{k+0} \) in curving conditions:

\[
\left( \sigma_{k+0} \right)^2 = \left[ \frac{2\pi R}{n_0} \cdot n_k \cdot \cos(\alpha_k + \alpha_0) \right]^2 \cdot \sigma_{k+0}^2 + \left[ \frac{2\pi R}{n_0} \cdot n_k \cdot \sin(\alpha_k + \alpha_0) \right]^2 \cdot \sigma_R^2
\]

(18)

and by means of \( \sigma_{G(\nu_k)} \) it was possible to estimate \( \sigma_{G(\nu_k)} \) in curving conditions:

\[
\left( \sigma_{G(\nu_k)} \right)^2 = T_{C} \cdot \sigma_{G(\nu_k)}
\]

(19)

During curving the uncertainties that were estimated on the mock-up of the industrial robot described in the following paragraph were: \( \sigma_R = 0.1\cdot 10^{-3} \) m; \( \sigma_{k+0} = 0.0126 \) rad; \( \sigma_{l} = 0.0025 \) rad/s (the calculations were evaluated at the nominal velocity and at the nominal steering angle). The ratio of the standard uncertainties (eq. 18 and 19) was computed to estimate the uncertainty ratio \( \xi_k \) roughly equal to 2. From practical considerations this ratio can be considered constant even when changing velocity or the steering manoeuvre.

3.4 Steering

During steering the relative accuracy of the gyroscope is higher (more than in curving) than the relative accuracy of the encoder-based estimation, which suffers from wheel deformations. Driver wheel deformation during steering affects the odometric accuracy because of the wheel torsion that causes the actual heading to be different from the steering encoder output (the real heading is delayed with respect to the steering angle command). To show this effect, the ratio between the odometric and the inertial attitude increments and the steering angle are displayed in figure 4 as a function of time. When the steering velocity is low or zero, the two estimates have very similar mean values thus leading to a ratio near unity. When the steering velocity is different from zero, i.e. the vehicle is actually steering, the odometric estimates are different from gyroscope estimates: they are higher as the steering angle increases and lower as it decreases. It is worth noting that this effect, which could be thought of as systematic and constant, actually depends on temperature, friction, wear and age. Therefore, it was not characterised [16].
The accuracy ratio depends on the steering angle velocity and tends to compensate when steering on the right and then on the left. In order to estimate the standard uncertainty of $\sigma_{\alpha_k + \alpha_0}$ and $\sigma_{G(V_k)}$, the path depicted in figure 5 was followed 20 times; the attitude was estimated at the start and finish using the reference method and the odometric and inertial navigation algorithms. In order to avoid the compensating effect, the steering was carried out 10 times abruptly on the right and then each time slowly on the left, then vice versa 10 times. By computing the differences with respect to the laser reference system, the standard deviations $\sigma_{E_{\Delta t} - \Delta t}$ and $\sigma_{G(V_k)}$ were computed.

Taking into account the third equation of eq. 1 it was then possible to estimate $\sigma_{\alpha_k + \alpha_0}$ and $\sigma_{G(V_k)}$:

$$\sigma_{\alpha_k + \alpha_0} = \left( \frac{2\pi}{n_0} \cdot \frac{R}{b} \sum_{k=1}^{N} \left( \eta_k \cdot \cos(\alpha_k + \alpha_0) \right) \right)^{-1} \cdot \sigma_{E_{\Delta t} - \Delta t}$$

(20)

$$\sigma_{G(V_k)} = (N \cdot T_C)^{-1} \cdot \sigma_{G_{\Delta t}}$$

(21)

by means of $\alpha_{\Delta t}$ and $\alpha_0$ it was possible to estimate $\sigma_{E_{\Delta t}}$ in steering conditions:

$$\left( \sigma_{E_{\Delta t}} \right)_k = \left[ \left( \frac{2\pi}{n_0} \cdot \frac{R}{b} \cdot \cos(\alpha_k + \alpha_0) \right)^2 \cdot \sigma_{\alpha_k + \alpha_0}^2 + \left( \frac{2\pi}{n_0} \cdot \frac{R}{b} \cdot \sin(\alpha_k + \alpha_0) \right)^2 \cdot \sigma_R \right]^{1/2}$$

(22)

by means of $\sigma_{G(V_k)}$ it was possible to estimate $\sigma_{G_{\Delta t}}$ in steering conditions:

$$\left( \sigma_{G_{\Delta t}} \right)_k = T_C \cdot \sigma_{G(V_k)}$$

(23)

During steering the uncertainties that were estimated on the mock-up of the industrial robot described in the following next paragraph were: $\sigma_R = 0.1 \cdot 10^{-3}$ m; $\alpha_{\Delta t} = 0.0251$ rad; $\sigma_\alpha = 0.0025$ rad/s (the calculations were evaluated at the nominal velocity and at the nominal steering angle and steering velocity). The ratio of the standard uncertainties (eqs. 22 and 23) was computed to estimate the uncertainty ratio $\xi_R$ roughly equal to 4. From practical considerations this ratio can be considered constant even when changing velocity or the steering manoeuvre.

3.5 Summary of the uncertainty ratio

The accuracy ratio as a function of the vehicle manoeuvre is summarised in table 1. Qualitative considerations are also shown.

It is important to note that the technique adopted here simplified the one used in [8] for two reasons: first of all, here the algorithm had to be implemented on a real time system and therefore its computational weight had to be kept low; secondly the aim of this work was not to fully optimise the algorithm, but rather to show that the principle of data fusion presented here can be used successfully in the case of two relative systems applied to the navigation of Autonomous Guided Vehicles.

For the same reason the accuracy ratio was estimated for the four manoeuvres not taking into account the steering angle and the velocity. However, if more accurate results have to be achieved, these two parameters could be taken into account during the calibration procedure.

<table>
<thead>
<tr>
<th>Vehicle manoeuvre</th>
<th>Odometric accuracy: qualitative considerations</th>
<th>Inertial accuracy: qualitative considerations</th>
<th>Accuracy ratio $\xi_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standing</td>
<td>High</td>
<td>Low</td>
<td>0</td>
</tr>
<tr>
<td>Curving with the</td>
<td>Medium</td>
<td>High</td>
<td>2</td>
</tr>
</tbody>
</table>
4. Experimental results

In the first part of this section the effectiveness of the data fusion algorithm is shown by comparing its estimation with the one obtained by odometers or by gyroscope attitude estimation. The paths achieved by encoder attitude increments only ($\Delta \delta_k = \Delta \delta_k^E$ in equation 5), by gyroscope attitude increments only ($\Delta \delta_k = \Delta \delta_k^G$ in equation 5) and by data fusion (equation 5 with the accuracy ratio of table 1) were computed. The second part of this section then presents the estimation of equi-uncertainty ellipses as described in equation 9 and 10. The ellipses give the position error estimation within a confidence level of twice the standard deviation and are computed as a function of the actual measured path. In order to verify the proposed technique experimentally, a mock-up of a three-wheeled industrial robot about 600 mm long, 300 mm wide and 400 mm tall (see figure 6) was used. The driver-steering wheel, driven by two DC motors, is the front wheel. Two incremental encoders and a fibre-optic gyro were mounted on the vehicle. The driver encoder has 625 ppr, the steering encoder 10000 ppr and the fibre-optic gyro a nominal sensitivity of 55 [mV/°/s]. The control system and navigation equations were computed by an industrial PC embedded in a PXI system together with an analogue and a digital board. The analogue board reads the gyroscope output and feeds the reference velocity for the driving and the steering axes to a control axis board that drives the motors. The digital board reads the encoders output and computes the increments.

In order to verify the data-fusion algorithm, a triangulation sensor based upon an optical scanner mounted on the robot and infrared transmitters fixed in the environment [3] were used. The angular uncertainty is ±47 arc-seconds, and the position accuracy in the centre of a 5m square room is ±1 mm (confidence level ±2σ).

Twenty rectangular paths in both clockwise and counter-clockwise directions at the velocity of 0.25 m/s were carried out as shown in figure 7. These paths were followed keeping the robot load and the current fed to the motors constant (i.e. following the same trajectories in terms of steering and velocity). The initial and end positions were estimated using the absolute triangulation system and then compared with the odometric, gyroscope and data-fusion estimates (fig. 8). The distances between the end point estimates of the three relative navigation systems and the reference triangulation system represent the estimated errors. In the path shown, as is the case for each final point estimation, the improvement given by the proposed technique can clearly be seen (fig. 8).

In figure 9 the histogram of the estimated end-point errors for the three navigation errors is shown. A binormal distribution of the estimated errors can be seen which is due to a lower accuracy on the counter-clockwise path with respect to the clockwise path. This could be explained by taking into account the reset phenomena of the zero angle (auto-zero implemented in the digital board) which is slightly different when the steering axis encounters the zero from the clockwise or counter-clockwise directions, and/or by taking into account the residuals on calibration [15, 16].

The mean absolute value of the estimated error in distance over the 25 meter long path was 49 mm by odometric navigation and 28 mm by odometric plus inertial attitude navigation, while it was 15 mm by data fusion between the odometric and the inertial attitude estimations. It is worth noting that the data fusion final-point estimation gave the best result for each path.

<table>
<thead>
<tr>
<th>Vehicle status</th>
<th>$n_k$</th>
<th>$\alpha_k$</th>
<th>$(\alpha_k - \alpha_{k,i})/T_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standing</td>
<td>0</td>
<td>any</td>
<td>any</td>
</tr>
<tr>
<td>Curving with the steering</td>
<td>0</td>
<td>$\geq 10^\circ$</td>
<td>$&lt; 1^\circ$/s</td>
</tr>
<tr>
<td>angle blocked</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steering</td>
<td>0</td>
<td>any</td>
<td>$\geq 1^\circ$/s</td>
</tr>
<tr>
<td>Straight path</td>
<td>0</td>
<td>$&lt; 10^\circ$</td>
<td>$&lt; 1^\circ$/s</td>
</tr>
</tbody>
</table>

Tab 2. Definition of the different vehicle manoeuvres as a function of encoder data.
Figure 10 shows the uncertainty ellipses caused by the systematic effects discussed in § 2 as a function of the path (the ellipses are generated taking into account the first two rows and the first two columns of the matrix $C_{\chi k}$, see equation 9). The correlation increment at the end of the path and in general on the left side of the path can be seen. The correlation increment that gives a thin ellipse (i.e. a preferred direction) is caused by the fact that the influence parameters give rise to the same direction in end-point estimation errors. This could be easily verified by computing the end-point estimation varying each influence parameter one at a time around the nominal value within twice the standard uncertainty: the end-point estimations of all the influence parameters give rise to roughly the same geometrical figure (pointing in the same direction).

To show the effectiveness of the proposed algorithm in evaluating the uncertainty ellipses, a set of 4 paths was repeated changing the vehicle load and velocity: on the first path the vehicle had a nominal load and a velocity equal to 0.25 m/s; on the second the vehicle had the nominal load plus 10 kg and a velocity equal to 0.25 m/s; on the third the vehicle had a nominal load and a velocity equal to 0.5 m/s; on the fourth the vehicle had the nominal load plus 10 kg and a velocity equal to 0.5 m/s. Figure 11 shows the end-point estimation differences between the proposed algorithm and the reference system and the estimated uncertainty ellipse computed with a probability of twice the standard uncertainty considered to have a normal distribution.

Figure 12 shows a zoom on path reconstruction and uncertainty ellipses as a function of a straight path. There is a huge variation in the geometry and orientation of the ellipses as a function of path. This clearly shows the need for the covariance estimation of uncertainty and not only the standard uncertainties $\sigma_x$ and $\sigma_y$, which could lead to inaccurate fusion with an absolute system (for example with laser scanner position estimation).

5. Conclusions

This paper presents a description of a ‘sensor fusion’ algorithm for a navigation system that uses the output of two relative navigation systems: an odometric one based on encoders and an inertial one based on a gyroscope. The accuracy of the two navigation systems was estimated as a function of the actual manoeuvre which was identified by the navigation data itself, then the output data were combined taking into account the previously calibrated accuracy ratio. The parameters of the navigation models were calibrated as a function of the different manoeuvre by using a scaled mock-up of a three wheeled industrial robot.

In order to calibrate the proposed technique, the mock-up followed 25-meter long rectangular path in both clockwise and counter-clockwise directions. The initial and end positions were estimated using a reference absolute triangulation system. The mean estimated error in distance over the path was found to be 49 mm by odometric navigation and 28 mm by odometric plus inertial attitude navigation, while it was found to be 15 mm by data fusion between the odometric and inertial attitude estimations. It worth pointing out that the data fusion final-point estimation gave the best result for each path.

A recursive method for the estimation of the evolution of spatial uncertainty was developed taking into account the systematic effects as well. Calibration of the uncertainty parameters was carried out as a function of the different
The uncertainty ellipses were thus computed as a function of the path. The estimated errors in the end point measurement was carried out using the reference system with different loads and speed conditions. The compatibility with the uncertainty ellipses was verified.

**List of symbols**

\( (x,y) \) the sensor fusion estimated position with respect to the fixed reference of the reference point \( P_r \) on the vehicle

\( (x',y') \) the driver wheel encoder estimated position with respect to the fixed reference of the reference point \( P_r \) on the vehicle

\( X_\Phi \) \( (x,y,\delta) \) position and attitude vector

\( C_V \) covariance matrix of the vector \( V \)

\( S_V \) matrix of standard deviations of the vector \( V \)

\( R \) the driver wheel radius

ICR Instantaneous Centre of Rotation

\( \alpha \) the steering angle when the ICR is at the infinity

\( \alpha_0 \) the steering angle with respect to \( \alpha_0 \)

\( n_0 \) the number of counts from the driving encoder

\( n \) the number of counts from the driving encoder in one turn

\( \delta(t) \) the vehicle with respect to the fixed reference

\( \delta^s(t) \) the encoder estimated attitude of the vehicle with respect to the fixed reference

\( \delta^g(t) \) the gyro estimated attitude of the vehicle with respect to the fixed reference

\( b \) the distance between the rotation axis of the driver wheel and the axis of the back wheel which leads the manoeuvre

\( V_g(t) \) the gyro voltage output

\( T_e \) the sampling period

\( G_i(t) \) the gyro characteristic

\( DF(t) \) the algorithm of Data Fusion

\( d\Phi \) Jacobian of the vector function \( \Phi \)

\( \lambda \) the standard uncertainty in parameter \( \lambda \)

\( \epsilon \) the uncertainty in parameter \( \lambda \) defined with a coverage factor of two

### 6. References


From equations 1, 4 and 6 it is possible to write:

\[
\begin{align*}
    x_{k+1} &= \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \\
    y_{k+1} &= \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \sin(\delta_i) \\
    \delta_{k+1} &= \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \sin(\alpha_i + \alpha_0) \cdot \frac{1}{b} \cdot \frac{T_C \cdot G_i}{\xi_R^2 + 1} + \sum_{i=0}^{k} \frac{T_C \cdot G_i}{\xi_R^2 + 1}
\end{align*}
\]

From the above it is possible to consider the uncertainties of the \( \varepsilon_{wk}(i) = [\varepsilon_R, \varepsilon, \varepsilon_G, \varepsilon_k] \). The following is obtained by linearising the above equation with respect to \( \varepsilon_{wk}(i) \):

\[
\begin{align*}
    \varepsilon_{xk+1} &= \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \varepsilon_R - \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \varepsilon_k - \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \sin(\delta_i) \cdot \varepsilon_k \\
    \varepsilon_{yk+1} &= \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot \cos(\alpha_i + \alpha_0) \cdot \sin(\delta_i) \cdot \varepsilon_R - \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \sin(\delta_i) \cdot \varepsilon_k + \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \varepsilon_k \\
    \varepsilon_{\delta k+1} &= \sum_{i=0}^{k} \frac{T_C \cdot G_i}{\xi_R^2 + 1} + \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot \frac{1}{b} \cdot \sin(\alpha_i + \alpha_0) \cdot \varepsilon_R + \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot \frac{1}{b} \cdot \cos(\alpha_i + \alpha_0) \cdot \varepsilon_R
\end{align*}
\]

At this point it is possible to estimate the standard uncertainty \( \sigma_{xk+1} \) of the \( x_{k+1} \) component by taking the expected value of the squared \( \varepsilon_{xk+1} \). By taking into consideration the hypothesis discussed in § 2:

\[
\begin{align*}
    \mathbb{E}\{ \varepsilon_{wk}(i) \cdot \varepsilon_{wk}(j) \} &= 0 \quad \forall \ i, j, \forall \gamma, \forall \mu \\
    \mathbb{E}\{ \varepsilon_{wk}(i) \cdot \varepsilon_{wk}(i) \} / (\sigma_{wk}(i) \cdot \sigma_{wk}(i)) &= 1 \quad \forall \ k, \forall \ h
\end{align*}
\]

it is possible to write:

\[
\sigma_{xk+1}^2 = \left( \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \sigma_R^2 + \left( \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \sin(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \sigma_k^2 \right)^2 + \left( \sum_{i=0}^{k} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \sin(\delta_i) \cdot \sigma_R^2 \right)^2 \right)^2
\]

which can be rewritten in the following way:

\[
\sigma_{xk+1}^2 = \left( \sum_{i=0}^{k-1} \frac{2\pi}{n_0} \cdot n_i \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) + \frac{2\pi}{n_0} \cdot n_k \cdot \cos(\alpha_k + \alpha_0) \cdot \cos(\delta_k) \right)^2 \cdot \sigma_R^2 + \\
\left( \sum_{i=0}^{k-1} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \sin(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \sigma_R + \frac{2\pi}{n_0} \cdot n_k \cdot R \cdot \sin(\alpha_k + \alpha_0) \cdot \cos(\delta_k) \cdot \sigma_k \right)^2 + \\
\left( \sum_{i=0}^{k-1} \frac{2\pi}{n_0} \cdot n_i \cdot R \cdot \cos(\alpha_i + \alpha_0) \cdot \sin(\delta_i) \cdot \sigma_R + \frac{2\pi}{n_0} \cdot n_k \cdot R \cdot \cos(\alpha_k + \alpha_0) \cdot \sin(\delta_k) \cdot \sigma_R \right)^2
\]
therefore,

\[ \sigma_{x_{k+1}}^2 = \sigma_{x_k}^2 + \left( \frac{\Delta x_k}{\partial R} \right)^2 \cdot \sigma_R^2 + 2 \cdot \frac{\partial \Delta x_k}{\partial \alpha_k} \cdot I_{\alpha k} \cdot \sigma_{\alpha_k} + \frac{\partial \Delta x_k}{\partial \delta_k} \cdot \sigma_{\delta_k}^2 + 2 \cdot \frac{\partial \Delta x_k}{\partial \delta_k} \cdot I_{\delta k} \cdot \sigma_{\delta_k} \]

where

\[ I_{\alpha k} = \sum_{i=0}^{k-1} \frac{2\pi}{n_0} \cdot n_i \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \sigma_R \]

\[ = \sum_{i=0}^{k-1} \frac{2\pi}{n_0} \cdot n_i \cdot \cos(\alpha_i + \alpha_0) \cdot \cos(\delta_i) \cdot \sigma_R + \frac{2\pi}{n_0} \cdot n_{k-1} \cdot \cos(\alpha_{k-1} + \alpha_0) \cdot \cos(\delta_{k-1}) \cdot \sigma_R \]

\[ I_{\delta k} = I_{\delta k-1} + \frac{\partial \Delta x_{k-1}}{\partial \delta_{k-1}} \cdot \sigma_{\delta_{k-1}} \]

in a similar way \( I_{\delta k} \) and \( I_{\delta_{k-1}} \) are defined recursively:

\[ I_{\alpha k} = I_{\alpha k-1} + \frac{\partial \Delta x_{k-1}}{\partial \alpha_{k-1}} \cdot \sigma_{\alpha_{k-1}} \]

\[ I_{\delta k} = I_{\delta k-1} + \frac{\partial \Delta x_{k-1}}{\partial \delta_{k-1}} \cdot \sigma_{\delta_{k-1}} \]

By developing the other terms of \( C_{x_{k+1}} \) in a similar way it is possible to resume the matricial form of eq. 9:

\[ C_{x_{k+1}} = C_{x_k} + \mathcal{Z}_{\Phi_k} \cdot C_{w_k} \cdot \mathcal{Z}_{\Phi_k}^T + \mathcal{Z}_{\Phi_k} \cdot S_{w_k} \cdot I_k^T + I_k \cdot S_{w_k} \cdot \mathcal{Z}_{\Phi_k}^T \]

\[ I_k = I_{k-1} + \mathcal{Z}_{\Phi_{k-1}} \cdot S_{w_{k-1}} \]

where the terms are defined as follows:
and where \( \sigma_k \) has to be updated at each step in the matrix \( C_{\text{sk}} \) after \( C_{\text{Xk}} \) has been calculated.

\[
C_{\text{Xk}} = \begin{bmatrix}
\sigma_{xk}^2 & \sigma_{xy} & \sigma_{x\theta} \\
\sigma_{xy} & \sigma_{yk}^2 & \sigma_{y\theta} \\
\sigma_{x\theta} & \sigma_{y\theta} & \sigma_{\theta k}^2
\end{bmatrix}
\]

\[
C_{\text{sk}} = \begin{bmatrix}
\sigma_k^2 & 0 & 0 & 0 \\
0 & \sigma_{\alpha k}^2 & 0 & 0 \\
0 & 0 & \sigma_G^2 & 0 \\
0 & 0 & 0 & \sigma_{\theta k}^2
\end{bmatrix}
\]

\[
S_{\text{sk}}(i,j) = \sqrt{C_{\text{sk}}(i,j)} \quad \forall i, \forall j
\]

\[
\mathcal{A}_{\phi k} = \begin{bmatrix}
\frac{\partial \nabla x_k}{\partial R} & \frac{\partial \nabla x_k}{\partial \alpha_k} & \frac{\partial \nabla x_k}{\partial G} & \frac{\partial \nabla x_k}{\partial \delta_k} \\
\frac{\partial \nabla y_k}{\partial R} & \frac{\partial \nabla y_k}{\partial \alpha_k} & \frac{\partial \nabla y_k}{\partial G} & \frac{\partial \nabla y_k}{\partial \delta_k} \\
\frac{\partial \nabla \delta_k}{\partial R} & \frac{\partial \nabla \delta_k}{\partial \alpha_k} & \frac{\partial \nabla \delta_k}{\partial G} & \frac{\partial \nabla \delta_k}{\partial \delta_k}
\end{bmatrix}
\]

\[
I_k = \begin{bmatrix}
I_{X\alpha k} & I_{XGk} & I_{X\theta k} \\
I_{Y\alpha k} & I_{YGk} & I_{Y\theta k} \\
I_{\alpha \alpha k} & I_{\alpha Gk} & I_{\alpha \theta k}
\end{bmatrix}
\]
Captions

Fig. 1 Kinematics scheme of the three wheeled AGV. The attitude $\delta$ is the angle between the absolute reference system $xOy$ and the mobile reference system $XP_Y$. The position $Pr$ is defined by the vector $(x,y,\delta)$ which takes into account the attitude of the mobile robot.

Fig. 2 Gyroscope increments in scaled values for representation purposes and driving wheel velocity as a function of time during the two rectangular revolutions shown in figure 7.

Fig. 3 Gyroscope output increments during straight path at 16 ms integration time (upper figure) and gyroscope output FFT (lower figure).

Fig. 4 Encoder versus gyroscope increments and steering angle during one 90° turn.

Fig. 5 Vehicle path for uncertainty ratio estimation during steering.

Fig. 6 Mock-up of the three wheel industrial robot used for experimental verification.

Fig. 7 Rectangular path reconstructed by pure odometric, odometric plus inertial and data fusion navigation. Velocity was 0.25 m/s.

Fig. 8 Zoom on end-point estimation by pure odometric, odometric plus inertial and data fusion navigation. The end point was estimated using the absolute measurement system.

Fig. 9 Histogram of the absolute value in final position error estimation by pure odometric, odometric plus inertial and data fusion navigation.

Fig. 10 Path reconstruction using the data-fusion algorithm with uncertainty ellipses drawn as a function of the path.

Fig. 11 Path estimation using Data Fusion (continuous line); final point estimation using the reference system (+); final point displacements estimated using the reference system with respect to the end point estimation by Data Fusion (*).

Fig. 12 Zoom on a path reconstruction made using the data-fusion algorithm with uncertainty ellipses drawn for the path achieved in the second turn.