1. INTRODUCTION

The hot wire anemometer has been used for many years as a research tool in fluid mechanics. In spite of the introduction of new velocity measurement systems (i.e. the laser Doppler velocimeter), its applications are still expanding, largely due to improvements of electronic technology and to increased interest in detailed description of turbulent flow fields. Furthermore, the hot wire anemometer is still the only instrument delivering at the output a truly analogue representation of the velocity field up to high frequencies fluctuations.

The hot wire anemometer consists of a sensor, a small electrically heated wire exposed to the fluid flow and of an electronic equipment, which performs the transformation of the sensor output into a useful electric signal. Contrary to most measuring instruments, the electronic circuitry forms an integral part of the anemometric system and has a direct influence on the probe characteristics.

The sensor itself is very small : typical dimensions of the heated wire are 5 μm in diameter and 1 to 3 mm in length, thus giving an almost punctual measurement volume.

The basic principle of operation of the system is the heat transfer from the heated wire to the cold surrounding fluid, heat transfer which is function of the fluid velocity. Thus, a relationship between the fluid velocity and the electrical output can be established.

The purpose of the electronic circuit is to provide to the wire a controlled amount of electrical current, and in the constant temperature method, which is by far the most used type of operation, to vary such a supply so as to maintain the wire temperature constant, when the amount of heat transfer varies.

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COURSE NOTE 117

HOT WIRE TECHNIQUES

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2. PROBES

2.1. Hot wire sensors

In practical applications, a material suitable for making a sensor should have some properties :

1 - a high value of the temprature coefficient of resistance, to increase its sensitivity to velocity variations ;

- an electrical resistance such that it can be easily heated with an electrical current at practical voltage and current levels ;

2 - STABILITÀ PROPRIETÀ SUPERFICIALE (NO OSSIDAZIONE)

- possibility of being available as wire of very small diameter ;

3 - a high enough tensile strength to withstand the aerodynamic stresses at high flow velocities.

The materials which are most commonly used are : tungsten, platinum, and platinum-iridium alloys. Tungsten wires are mechanically strong and have a high temperature coefficient of resistance (0.004/°C). However, they cannot be used at high temperatures in many gases, because of their poor resistance to oxidation. Platinum has good oxidation resistance, has a good temperature coefficient (0.003/°C), but is mechanically weak, particularly at high temperatures. The platinum-iridium alloy is a compromise between tungsten and platinum with good oxidation resistance, and higher tensile strength than platinum, but it has a low temperature coefficient of resistance (0.00085/°C). Platinum-rhodium alloy has a slightly higher temperature coefficient than platinum-iridium, but is not as strong mechanically.

	TUNGSTENO	PLATINO	PLATINO-IRIDIO
1	ok	ok	
2		ok	ok
3	ok		ok

To coat := coprire

Tungsten is presently the most popular hot wire material. When coated with a thin platinum layer, it becomes more resistant to oxidation and soldering to the support needles is eased.

The actual tendency is to use a tungsten wire (or platinum coated tungsten wire) coated at the extremities by a thick copper or gold deposit (Fig. 1). This is soldered to the shaped support and the resulting probe has good mechanical and aerodynamic characteristics. Typical dimensions are 1 to 10 μm in diameter, 5 μm being the most used choice, and a length of 1 to 3 mm for the heated wire.

2.2 Hot film sensors

The hot film sensor is essentially a conducting film laid on a ceramic or quartz substrate. The sensor can be, typically, a quartz rod coated with a platinum film. When compared with hot wires, the cylindrical hot film sensor has many advantages, the first being that it is less susceptible to fouling and easier to clean. However, the double structure - film plus substrate - makes its frequency response more complex.

The metal film thickness on a typical film sensor is less than 1000 Å. Thus, the mechanical strength and the effective thermal conductivity of the sensor are determined almost entirely by the substrate material. Most films are made of platinum due to its good oxidation resistance and the resulting long term stability : its low strength is not a drawback since it is supported by the substrate. Because of their ruggedness and stability, film sensors have been used for many measurements which were otherwise very difficult with the more fragile and less stable hot wires. Some manufacturers utilize special film coating techniques. For measurements in air, hot films are coated with a thin layer of high purity alumina. This is highly resistant and has high thermal conductivity to minimize loss of frequency response. Hot film sensors made for water or conducting fluids have a heavier coating of fused quartz which provides complete electrical insulation.

Even though film sensors have basic advantages, hot wire sensors give superior performance in many applications. The diameter of a cylindrical film sensor is typically 0.025 mm or larger. When compared with the dimensions of a hot wire, this is quite large. In addition, the film sensor generally has a lower sensitivity. Therefore, in application requiring maximum frequency response, minimum noise level and very close proximity to a surface, the tungsten hot wire sensor is superior.

NOTA

3. CALIBRATION CHARACTERISTICS

Figure 1 shows a typical hot wire probe of 0.005 mm in diameter and figure 2 its velocity calibration curve in an air stream. The calibration curve is non-linear, with maximum sensitivity at low velocities. The disadvantages of a non-linear output in terms of convenient reading and recording of mean and fluctuating velocity components are well known. The reason for this behaviour is given by the relation for the heat transfer from a body in a flowing air stream.

Writing such an equation for the hot wire, if I is the heating current flowing in the wire, R_W is the resistance at operating temperature T_W, d and ℓ its diameter and length, the steady state energy balance takes the form :

$$\text{Supply} = I^2 R_W = \varepsilon \pi d \ell \alpha \, (T_W - T_a) = \text{dissipation}$$

where α is the heat transfer coefficient and ε unities conversion coefficient. α is related to the other thermodynamic properties of the fluid in the Nusselt number :

$$Nu = \frac{\alpha d}{K}$$

where K is the thermal conductivity coefficient for the fluid.

By substitution, we obtain :

$$I^2 R_W = \varepsilon \pi \ell K \, (T_W - T_a) \; Nu$$

The problem is now to obtain a relation between the Nusselt number and the other thermodynamic properties of the fluid and the characteristics of the flow around the thin wire. Typically, by dimensional analysis one must have :

$$Nu = f(Re, Pr)$$

where Re = Reynolds number $\frac{V \cdot d}{\nu}$

Pr = Prandtl number $\frac{C_p \mu}{K}$

The determination of the above relation has been the subject of many investigations trying to determine a universal cooling law for cylinders. Some of the results are shown in Table 1 and one example is given in figure 3 . They obviously fail the main purpose of establishing a universal law, but are very useful in indicating the trends where changing the fluid properties. It should be noted that the Prandtl number does not appear in these relations because they are established for air at ambient temperature (indicated as T_g in the tables. T_m is defined as $\frac{T_w+T_g}{2}$). The most widely accepted law is that of Collis and William.

It is easy to see that most of these relations are of the kind $A+BV^n$ so the energy balance equation may be rewritten simply as :

$$I^2 R_w = |A+BV^n| \; (T_w-T_a) = H(V) \; (T_w-T_a) \qquad (1)$$

Obviously, A represents the natural convection term and BV^n the forced convection term.

NOTA:

Sensors can be manufactured to high accuracy standards so that the A and B values are very close for wires coming from the same batch : not so close, however, to dispense altogether from the direct calibration of each wire if a consistent accuracy of results is needed. Taking into account the **uncertainty of the values of A, B** and **n**, and the fact that the problems are made even worse by the **finite length of the wire** and the **effect of the support**, one may say that a direct calibration of the actual probe has to be made every time before a test. Furthermore, because of the dependence on fluid properties, the calibration has to be made in the same fluid at the same temperature as for the actual tests.

To stem [uguale] := bloccare

A further point which stems from the above considerations is that eq. 1, with constant coefficients, is not directly valid when the wire is operated in constant current mode (Ref. 3) because changes in fluid properties occur as the temperature of the wire changes following variation of flow velocity.

The output contains information both for the natural convection and for the forced convection. If one wants to increase the resolution of the measurements, the influence of the natural convection term should be minimized. Good working conditions are reached when :

$$\frac{1}{d}\sqrt{Gr} > 1$$

(or alternatively, $Re > Gr^{1/3}$, Ref. 3), Gr being the Grashof number $= \frac{\beta g (T_w - T_w) d^3}{\nu^2}$

Relation (1) could be practically exploited for measurements if a suitable mean is found to extract an information about the actual value of the heat transfer. This can be done easily because the resistance of a wire is a function of its temperature. For a metallic conductor :

$$R_w = R_a\,[1 + b_1(T_w - T_a) + b_2(T_w - T_a)^2 + \ldots]$$

where the coefficients b_1 and b_2 have the following typical values

[illegible]

- platinum
  - $b_1 = 3.5 \cdot 10^{-3}/°C$
  - $b_2 = -5.5 \cdot 10^{-7}/°C^2$

- tungsten
  - $b_1 = 5.2 \cdot 10^{-3}/°C$
  - $b_2 = 7 \cdot 10^{-7}/°C^2$

It could be seen that for usual operating temperatures, i.e., up to around 200°C; the above expressions could be linearized to :

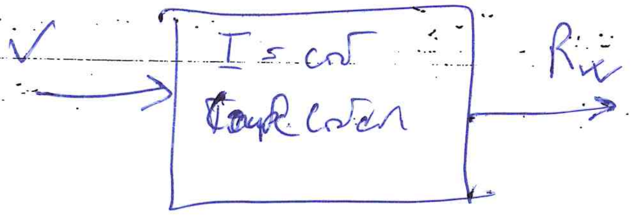
$$R_w = R_a \left| 1 + b_1 (T_w - T_a) \right| \Rightarrow T_w - T_a = \frac{R_w - R_a}{b_1 R_a} \Rightarrow$$

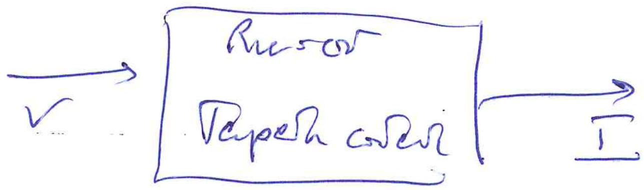
Upon substitution in (1) one obtains :

$$b_1 \frac{I^2 R_w R_a}{R_w - R_a} = A + BV^n \qquad (2)$$

Thus, **the information about the actual value of the heat transfer could be obtained either as the value of R_w if I is kept constant, or as the value of I if R_w is kept constant.** The two modes of operation are respectively known as constant current method and constant temperature method.

DUE MODALITA' DI LETTURA!





4. CONTROL CIRCUITS I - CONSTANT CURRENT

For a useful application of hot wire sensors, the selection of the probe is of primary importance; however, the sensor must be controlled by an electronic circuit to obtain the best possible performances.

In its basic form, the control circuit may be reduced to a source of constant current feeding a calibration and measurement bridge. Referring to figure 4, the two resistors R1 are chosen to be equal, the value of R is equal to the hot resistance of the wire (usually 1.8, the cold wire resistance) and the supply current increased until, for zero wind velocity, a balance is obtained at the bridge output. Any change in wind velocity will change the heat transfer, and thus the wire temperature and resistance and cause an unbalanced voltage to appear at the bridge output. This can be calibrated against flow velocity to obtain the wire calcibration curve, which will be similar to that of figure 2.

However, one of the major reasons for using a hot wire anemometer is its ability to detect and follow fast fluctuations of velocity. In the case of the circuit of figure 4, if the velocity changes takes place very rapidly, the response of the sensor will lag behind the actual change in velocity due to its own thermal inertia.

With reference to equation 1, the equation for the instantaneous heat transfer can be written as

$$I^2R = H(v)(T_w - T_a) + c\,\frac{dT_w}{dt} \qquad (3)$$

where **c represents the thermal inertia** of the wire itself

CAPACITA' TERMICA

$$c \cong k\,\frac{d^2\pi}{4}\,\ell .$$

Equation can be linearized and solved for small velocity perturbations, making

$$H(v,t) = \overline{H}(v) + h(v,t)$$

to obtain for the fluctuating output, defined as :

$$E(t) = \overline{E}(t) + e(t)$$

$$M \frac{de}{dt} + e = -h \qquad (4)$$

Equation 3 leads to the definition of the wire time constant M as

$$M \triangleq \frac{CR}{R_0 H}$$

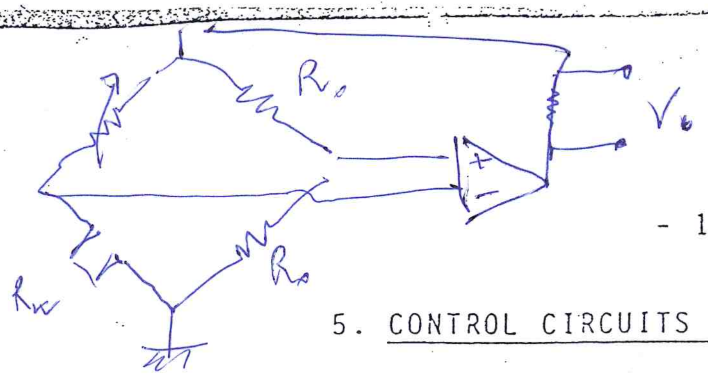
(where R is the hot wire resistance and R_0 the cold wire resistance) and of the **frequency limit** for the hot wire response to sinusoidal velocity fluctuations, as

$$f_0 = \frac{1}{2\pi M} \qquad (5)$$

Equations 4 and 5 show that the frequency response is function of the fluid velocity (through H), so that a universal value cannot be associated to these parameters. Typical values are **$f \cong 300$ Hz.**

Obviously, the above analysis is, and should be, restricted to the case of small perturbations, because in deriving equation 4 the effects of higher order, non-linear terms, have been neglected. The case of large fluctuations will not be dealt with here; a detailed account is given in reference 4 It should only be remembered that errors may become very important in the measurement of odd moments, i.e. skewness factor.

The frequency response is similar to that shown in figure 5. The attenuation of output with increase in frequency can be compensated for using an amplifier with gain increasing with frequency as in figure 6 (or because equation 3 is of the first order, a simple differentiating amplifier). Good results can be obtained, but the dependence of M from flow velocity means that a new compensation has to be made every time the mean velocity changes, making the system very difficult to use.



5. CONTROL CIRCUITS II - CONSTANT TEMPERATURE

Most of the above drawbacks can be eliminated by using the constant temerature mode of operation. In its simplest form, it can be considered, with reference to figure 4, as a system in which the output from the bridge is amplified and used to control the supply voltage such as to maintain the wire temperature constant. The circuit takes the form shown in figure 7, and **now the amplifier output** E, required to maintain the wire at a constant temperature **is a function of the flow velocity**. The wire temperature is again fixed by the choice of the resistance R of the bridge (usually 1.8 times the wire cold resistance). If the amplifier gain A is large enough, the bridge unbalance is given by

$$e \cong \frac{E}{A} \qquad (6)$$

and the wire is constantly kept at a fixed temperature (or at least its temperature fluctuations are $\cong \frac{1}{A+1}$ times smaller than in the constant current mode).

The calibration curve is substantially similar to that shown in figure 2 and all coefficients of equation 1 can be considered constants. Because the changes in temperature are now much smaller, the thermal inertia of the wire can be expected to play a minor role in determining the frequency response, but it cannot be said that the limiting frequency is simply increased by a factor $\frac{1}{A+1}$. In fact, the new frequencies to be considered are so high that the reactive phenomena taking place in the wire connection cables and in the amplifier must be taken into account and these increase the order of the response equation corresponding to (3).

If the system is reduced to the simple circuit of figure 8, a second order equation can be written in place of (3), which leads to a resonance frequency function of

$$\omega = f\left(\frac{R_W}{R_0}, H, E, M_1, M, A, L'\right)$$

where L' is a compensation inductance used to minimize the effects of M and M_1, and E, the amplifier offset voltage acts as a damping of the system.

This is of course a very sketchy analysis of the problem, again limited to small fluctuation. More detailed approaches can be found in reference 7, and it can be shown that a third order response equation provides better description of the system. Furthermore, for a hot film this becomes a combination of fifth and second order equations (Ref. 5).

In any case, the limiting frequency of the system can be improved by careful adjustment of L' and of the offset voltage E, but this may lead to an increased danger of self-maintained oscillations. By adjusting L' and E, it is possible to achieve an optimal regulation, corresponding to a critically damped system and thus to a maximally flat frequency response.

The usual approach to evaluate the result so obtained is by mean of the so called square wave test : a square wave test current is injected at the wire terminal and the response of the system monitored on an oscilloscope. Because of the feedback system involved, the resulting output is (or theoretically should be) equal to the derivative of the input signal and takes the form shown in figure 9. If the order of the system is higher than the second there is some difficulty in translating the information from the square wave test into frequency response of the hot wire. A comparison of the results obtained with the two techniques and of the good agreement obtained is given in reference 6. Two typical results, for hot wire and hot film; are shown in figures 10 and 11. This means that optimum adjustment could be obtained in a matter of minutes.

It should in any case be remembered that frequency response depends on H(v), that is on fluid velocity. Hence optimization of the anemometer response should be done for the mean flow velocity at which the probe is likely to operate, or in case of impulsive flows, for the mean value of velocity. Then, according to the above reference a good anemometer response will be achieved even for the case of large fluctuations. Frequency response flat in a range exceeding 30 KHz could be easily be obtained which means that response to typical subsonic turbulent fluctuations could be virtually considered as flat.

A final point to be mentioned is that because the temperature of the wire remains almost constant, all non-linearities introduced by thermal lag effect are substantially smaller and in most cases negligible (reference 4).

6. LINEARIZATION

The existence of a region of flat frequency response covering all the spectrum of frequencies of interest, allows the instantaneous response of the hot wire to be written, even for unsteady flows, in an algebraic form as :

$$E^2 = A + B(V)^n \qquad (8)$$

where A and B are constants obtained by a calibration. The result resumed by equation 8 is very useful because it makes a linearization of the response possible, that is by electronic means obtain an output which is linear function of velocity. If coefficients in equation 8 remain constant for all flow velocities, the above procedure could be carried out simply by using log and antilog amplifiers of variable gain and this is the basic working principle of any logarithmic linearizer. Its accuracy is mostly based on the constance of the coefficient n. This is generally true for velocities in the range 2 to 100 m/sec. Exceeding these limits, it is almost impossible to obtain a perfect linear correlation. The above linearizer is by far the most widely used. An alternative approach is the so called polynomial linearizer, which approximates the inverse of the response curve,

$$V = f(E) = \left(\frac{E^2-A}{B}\right)^{1/n} \qquad (13)$$

by a (usually) fourth order polynomial

$$V = a + bE^2 + cE^3 + dE^4 + fE \qquad (14)$$

whose coefficients are determined by best fit programs.

Again, equation 14 can easily be obtained by electronic means. The final result so obtained is usually a better approximation of the real response curve that can be obtained with the logarithmic types. Another advantage is that it is easier to obtain a higher frequency response with polynomial linearizers.

A more recent approach is of course to discard linearizers altogether and use an in-line digital computer for the analysis of the sampled data. This option is probably the best but it must still be considered as a much more expensive one and really justified only in special fields. Anyway, the use of linearizers together with constant temperature hot wire anemometers is nowadays so comman that, in what follows, we will consider this to be used in all cases. Thus the output could be considered each time as a linear function of fluid velocity.

A more important point is that yaw sensitivity is almost independent of flow speed if the Reynolds number is not too low (Ref. 3) so that a single calibration curve may be stored to be used in all the flow field, as shown in figure 14.

The angular sensitivity of the hot wire may be used to determine the direction and intensity of the velocity in an unknown two dimensional flow field. Making two measurements at α and $\alpha+\theta$, it is possible to write :

$$U_{eff_1} = E_1 = U\ f(\alpha)$$
$$U_{eff_2} = E_2 = U\ f(\alpha+\theta) \qquad (18)$$

where E_1 and E_2 are the two anemometer outputs. Thus

$$\frac{E_1}{E_2} = \frac{f(\alpha)}{f(\alpha+\theta)} = F(\alpha) \Rightarrow \text{to determine } \alpha . \qquad (19)$$

if θ is known.

$F(\alpha)$ can be obtained from the wire angular calibration curve $f(\alpha)$. Equation 19 allows the determination of the unknown angle α and (18) of the absolute velocity U.

To increase the accuracy a redundant measurement can be made at $(\alpha-\theta)$ to obtain :

$$E_3 = U\ f(\alpha-\theta)$$

and

$$\frac{E_3}{E_1} = \frac{f(\alpha-\theta)}{f(\alpha)} = F(\alpha) \qquad (20)$$

The only condition which must be satisfied is that the angles α, $(\alpha-\theta)$, $(\alpha+\theta) < 60°$, for the calibration curve to be valid. If these values are exceeded in the measurements, the results give an estimate of the true flow angle, which can be used to relocate the hot wire.

This can be extended to three dimensional measurements, figure 15, where the hot wire response is now function of U, α, β,

$$U_{eff} = F(U,\alpha,\beta) \tag{21}$$

Equation 21 can be rewritten as

$$U_{eff} = U\ f(\alpha)\ g(\beta) \tag{22}$$

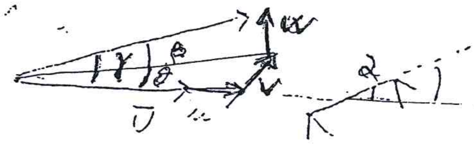
and used to determine the unknown flow direction in a three dimensional flow field when the two calibration curves $f(\alpha)$, $g(\beta)$ are known.

8. MEASUREMENT OF TURBULENCE

The directional sensitivity of the hot wire is also used to resolve the fluctuating components of the velocity field.

With reference to figure 16, the wire is located at an known angle α (and which can be determined as shown previously) in the plane of the mean velocity U. The instantaneous velocity U_I has the value :

$$U_I = \sqrt{(U+u)^2 + v^2 + w^2} \qquad (23)$$



and forms a solid angle γ with the mean velocity. If θ and β are the two projections of γ in the plane of the wire and in the plane normal to it, then :

$$U_{eff} = U_I \; f(\alpha+\theta) \; g(\beta) \qquad (24)$$

If the fluctuation components are small, β constitutes a small rotation around the wire and can be neglected, thus

$$E_I = U_{eff} = U_I \; f(\alpha+\theta) \qquad (25)$$

which developed in Taylor series gives

$$E_I = U_I \; f(\alpha) + U_I \left|\frac{\partial f}{\partial \alpha}\right|_\alpha \theta = U \; f(\alpha) + u \; f(\alpha) + v \; f'(\alpha) \qquad (26)$$

if the substitution $\frac{v}{U} = \theta$ is made and higher order terms are neglected. The above is a fairly crude linearization of the problem and shown just as a first order approach. For instance, retaining the higher order terms leads, for a wire normal to the mean velocity, that is $\alpha = 0$, to

$$U_I = U \left(1 + \frac{u}{U} + \frac{u^2}{2U^2} + \frac{w^2}{2U^2} + \frac{w^2}{2U^2}\right)$$

Then the mean output is given by the mean velocity plus the square mean values of turbulence components. This is often referred to as a hot wire non linearity. The implication of non linearities in the other quantities measured is important, but will not be dealt with here. Very exhaustive analysis can be found in reference 4.

Equation 26 may be used to determine the fluctuating velocity components : considering only its fluctuating part

$$e = u\, f(\alpha) + v\, f'(\alpha) \tag{27}$$

and taking its mean square value

$$\overline{e^2} = \overline{u^2}\, f^2(\alpha) + \overline{v^2}\, f'^2(\alpha) + \overline{uv}\, f(\alpha)\cdot f'(\alpha) \tag{28}$$

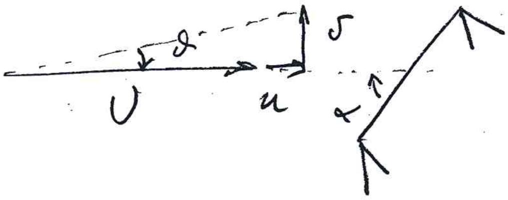
It is easy to see that making meaqurements at three angles α (in the place 1,2 of figure 16), it is possible to determine the values of $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$.

By having the probe in the plane 1,3 and repeating the procedure it is possible to determine $\overline{w^2}$ and $\overline{uw}$; that is in total 5 of the components of the Reynolds stress tensor.

By looking at the geometry of figure 12, it is possible to see that the 5 angles so required can be obtained by a conveniently slanted probe rotated around its axis (Ref. 8). It is thus theoretically possible to obtain a full survey of the mean velocity and of the turbulence field, once the angular characteristics of the probe are known. Again the accuracy of the measurements is increased by making redundant measurements, that is using more probe angles than strictly necessary.

A more common procedure is to use X-probes, that is double wire probes with the wires already fixed at predetermined angles on a single support. Then couples of measurements could be obtained without rotating probes and more important, simultaneously, so that more accurate data reduction techniques could be applied.

Rilevazione della Turbolenza



π : filo e vettore U definiscono tale piano.

Si nota quindi che una ulteriore componente w non modificherebbe l'orientazione relativa, ne' il modulo in termini lineari ⇒

$$U_{eff} = U_{incid} \cdot f(\alpha+\vartheta) \simeq (U+u)\left(f(\alpha) + \frac{\partial f}{\partial \alpha}\vartheta \right)$$

dove $\vartheta \cong d\alpha \simeq -\frac{v}{U}$ ⇒

$$U_{eff} = U f(\alpha) + f'(\alpha) v + u f(\alpha) + f'(\alpha)\frac{1}{U} u \cdot v \Rightarrow$$

2° ordine

considerando solo la parte fluttuante e facendone il valor quadratico medio:

$$\overline{U^2} = \overline{u^2} f^2(\alpha) + \overline{v^2} f'^2(\alpha) + 2\,\overline{uv}\, f(\alpha) f'(\alpha)$$

Dunque bastano 3 misure a tre diversi angoli per individuare le componenti $\overline{u^2}$, $\overline{v^2}$, $\overline{uv}$.

Spostando il sensore sugli altri due piani si determinerebbero tutte le componenti del TENSORE DI REYNOLDS: la sonda DEVE essere spostata in quanto si hanno 6 incognite.

9. SOME PRACTICAL PROBLEMS

There are a good number of reasons why a real hot wire anemometer does not and indeed could not, behave exactly as a theoretical one, as was more of less assumed to be in the previous chapters.

For our purpose here such reasons could be conveniently, even if somewhat arbitrarily, classified into the following categories : electronics, thermal, aerodynamics.

9.1 Problems due to electronic circuitry

Some of these reasons, namely those connected with the frequency response and stability of the feedback loop for constant temperature anemometers have already been discussed. Some others to be added are : the frequency response of the linearizer circuit. This part of the system has a highly non-linear amplitude response which poses a number of problems. If it is achieved by using log-, anti-log amplifiers, the results obtained are always a trade-off between stability and frequency response : as a result the frequency response of a linearizer is far more limited, usually, than the theoretical limit of the anemometer itself, and phase shift introduced at this stage may be far from negligible. Also, a certain amount of distorsion is introduced in the signal by the necessity of having a number of operational amplifiers driven to the limits of their possibilities. It is difficult to give figures of general applicability, but these can be worked out, or better, measured once the design of the electronic circuit is known, without problems. The important point is to be aware of their existence and of the ways of minimizing them. The thermal stability of such a linearizer could also be a problem for experiments of long duration. Minimal variations of the values of some components (in the input bias circuit and in the non-linear feedback devices) leads to important deviations from the theoretical amplitude response function. And things may be a lot worse if the similar temperature effects on the anemometer circuitry are allowed to appear at the linearizer input. (It is useful to remind that all these circuits are D.C. coupled so that instabilities spread very easily from one to the other). If insufficient care about this point is taken at the design stage and during actual use, results may become very easily useless. The polynomial type linearizers may have the edge (theoretically at least, because monolytic multipliers with fixed gain could be used) on this point, but this may be more than overcome by the increased complexing of setting required. A real advantage will be the use of digital linearizers, a solution which could be achieved if on-line digital computers for data processing is available.

Then there is the problem of noise. The heated wire at the input of the circuit is a good white noise source : fortunately its resistance is low, but the current flowing into it, by electronic standards is not small. The noise generated at this level (and in the other bridge resistors) may be larger than that of all the remaining electronic circuitry, making the long lived dispute between C.T and constant current anemometers on this point not very important today. An example of the noise measured at the output of a C.T. anemometer is shown in figure 17 which indicates a signal to noise ratio of db.

It is difficult to evaluate the evolution of this noise with flow velocities, the measurements requiring the availability of an absolutely stable and turbulence free flow. Theoretically it should increase but not too dramatically.

High frequency components could also be filtered out if the region of interest for the flow field is known; but a point worth of consideration is the possibility of generation of intermodulation at lower frequencies because of the nonlinear response of the linearizer.

Finally, as result of personal experience, it must be said that anemometer circuits at the limit of stability become excellent pick-ups of radio frequency signal, providing the experimenter with a cocktail of turbulence and the latest success of pop-music. It could be very entertaining but not is exactly what is expected.

9.2 Problems associated with heat transfer

The fact that for its operation the sensor must be at a higher temperature than the flow is a source of a number of practical problems.

First, and widely known, is the fact that while the wire is hot, it supports, of much higher mass, are at ambient temperature and as such act as heat sinks at the wire extremities. Hence the wire temperature is not, and cannot be, uniform over its length. The temperature distribution on a hot wire sensor has been carefully investigated by Champaigne (Ref. 9) for various flow conditions. For a wire normal to the mean flow direction, the result is easily obtained and can be expressed by the analytical relation

This means that the mean temperature is lower than the theoretical one and as a consequence, the sensitivity of the wire to velocity changes is reduced, a result which is often summarized by considering an active wire length smaller than the geometrical one to take into account the heat losses at extremities :

$$\ell_a = \ell_g - \ell_e .$$

This interpretation, while useful, can be misleading while considering the influence of wire length on its sensitivity to velocity fluctuations.

If the wire is not normal to the flow, the temperature distribution is no more symmetric (Ref. 9) showing a peak in the downstream region, and the amount of this skewness increases with increasing the angle of incidence. Such change is the main responsible for deviation of the wire angular response from a simple cosinusoidal law.

Because the actual temperature distribution is determined by the probe geometry such as wire length to diameter ratio, diameter and length of the supports, as well as by the thermal properties of the different elements of the probe (wire and support thermal conductivity and heat transfer coefficients) and the nature of the junction between the wire and the support, it becomes an almost impossible task to determine a unique law (at least a simple one) to predict the final results. As already mentioned, the only useful alternative is to make individual calibration for each probe angular response if an adequate degree of accuracy is to be achieved during the measurements.

A second point of interest stems from the consideration that if the heat transfer of the wire is partially conditioned by the heat conduction through the supports, then the unsteady behaviour of the probe (that is, the effect of velocity fluctuation on wire temperature) do not follow anymore the first order equation of section 3 , but becomes at least a two-pole system. Different investigations have been made on the subject, often referred to as "the support effect on hot wire unsteady response" and the figures 18,19,20 show the results obtained by Olivari (Ref. 10). While the effect is quite evident for constant current operation, it seems that for properly adjusted constant temperature anemometer, it may be practically neglected. Hence, another reason to prefer this second mode of operation. For this case different techniques to evaluate experimentally the actual response have been suggested by a number of authors.

A third problem is that of the "thermal wake" generated by the wire. While a number of suggestions have been made to take profit of this situation as for example for the measurement of velocity direction (Ref.11), for measurement of a tracer concentration (Ref. 12) and even for measurement of the transversal velocity fluctuations (Ref. 13), the fact remains that it may be a cause of concern when using multiple hot wires for conventional velocity measurements. This is the case for stream-wise velocity correlations, where the existence of a velocity and thermal wake is really a source of problems, and to some extent in the case of multiple wire probes.

FInally, another important problem arises for the dual sensitivity of the wire to both flow velocity and temperature. If temperature and velocity fluctuations occur simultaneously the hot wire velocity signal is contaminated by the concomitant temperature fluctuation. This may be the case in many flows like simulated geophysical flows, high subsonic and transonic flows, but it becomes of primary concern in the scope of this lectures, when temperature is used as a tracer to help in the study of turbulent/non turbulent boundaries and in general of the interface of turbulent structures. While the use of this passive contaminants enance the sensitivity of dectection techniques care has to be taken in the final interpretation of the results.

For a linearized hot wire anemometer the response equation to the temperature contaminated turbulence field is, for the fluctuating part :

$$e = \alpha u - \beta \theta$$

where $\alpha = \left(\frac{\partial E}{\partial U}\right)_T$ velocity sensitivity

$\beta = \left(\frac{\partial E}{\partial T}\right)_U$ temperature sensitivity .

E = mean linearizer output.

If it is assumed that the fluctuating output is due to velocity fluctuation alone, then

$e = \alpha u_m$ and

$u_m = u \frac{\beta}{\alpha} \theta$ = measured "contaminated" velocity or

$$\overline{u_m^2} = \overline{u^2} - 2\,\frac{\beta}{\alpha}\,\overline{u\theta} + \frac{\beta^2}{\alpha^2}\,\overline{\theta^2}.$$

Similar reasoning could be extended to correlations and spectral densities, with equivalent results, all indicating that correct measurement may only be obtained by simultaneous measurements of velocity and temperature.

In view of the above equation, the typical rule that the problem may be neglected if for an overheat ratio of 1.8 the temperature fluctuations are less than 0.5°C, may be considered with care because its validity depends on the velocity temperature cross correlation statistic values.

9.3 **Aerodynamic problems**

Under this heading we shall consider a few different aspects related to the nature of the probe-flow interactions.

The most evident and known problem for hot wire anemometry is the impossibility for the probe to detect the sense of the velocity vector. This is inherent to the nature of the probe, and while not limited to the hot wire alone, it is particularly unpleasant for a sensor designed for measurements in turbulent flows : in fact it makes impossible its use for a large class of situations, namely, every time a flow reversal is present or its presence is suspected. Such a situation may be more common than expected : apart from the cases of evident flow recirculation, regionsof mean-flow-reversal or of flows with zero mean velocity may be expected to exist sometimes in the intermittent region at the edge of jets and possibly near the wall during the bursting phenomenon in boundary layers.

Ponctual solution to this problem has been suggested and applied, but always leading to such a degree of complexity of probe design as to make impossible their widespread use especially in narrow or confined flow regions.

In summary, it is a situation to accept as such (it can be spotted by probability density distribution measurements), taking care to avoid its effects on the measurements. Apparently only frequency shifted laser doppler velocimeter may prove a valid alternative in this application.

A second problem is that of aerodynamic interference : of the support on the reading of the wire itself or between the different supports in multi-probe arrangements such as for space correlation measurements. This point is too detailed and too particular to be discussed here more than by mentioning the great amount of work done on it by Comte-Bellot (Ref. 4).

Also important is the effect of possible attenuation of fine-scale velocity fluctuations by the finite length of the wire. Such spatial averaging effects have been extensively treated by Wijngaard (Ref. 14) for single and crossed hot wires. Results could be analyzed either in terms of global effects on the measured RMS values as function of the wire length to turbulence microscale ratio, or more usefully in terms of one dimensional spectra, as shown in figure 21. It should be noted that in practically every situation this spatial average effect is the limiting factor for the response of the wire to velocity fluctuations, by far more critical than the frequency response of the electronic circuits of constant temperature bridges. But again the detailed analysis is too long and complex for the scope of this lecture and readers are referred to the above references.

A final point worth spending some time on is that of data interpretation : the output of a hot wire measurement represents the temporal variation of the signal at a fixed point in the flow field.

As most of the description of turbulence structure is best done in terms of its spatial or wave number behaviour, there is the problem of relating spatial variation of velocity from time dependent measurements. Or, in more general terms, to determine how to relate the anemometer signal to the velocity field. The easiest approach is to use the Taylor approximation in the form

$$\frac{\partial}{\partial t} = - U \frac{\partial}{\partial x}$$

where U is the mean velocity in the x-direction.
Then, in wave number form one obtains :

$$K_1 = \frac{2\pi f}{U}$$

which is known as the "frozen flow" approximation.
Experimental evidence suggests that such an approach is too simplist. To analyze it, and following Champaigne (Ref.15) one may consider a fixed system (x,t) and a moving one (ξ,t) with steady convection velocity V.
The in the moving frame

$\xi = x - Vt$ and

$$U'(\xi,t) = U(x,t) - V$$

if the two frames coincide at $t = 0$.
Making use of the conventional separation in mean and fluctuating velocity components gives :

$$\overline{U}'(\xi,t) = \overline{U}(x,t) - V \qquad u'(\xi,t) = u(x,t)$$

Then

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_x = \left.\frac{\partial u'}{\partial \xi}\right|_\tau \left.\frac{\partial \xi}{\partial t}\right|_x + \left.\frac{\partial u'}{\partial \tau}\right|_\xi \left.\frac{\partial \tau}{\partial t}\right|_x$$

if the flow pattern is frozen $\frac{\partial u'}{\partial \tau} = 0$, (or at least the time scale of the turbulent structures is long with respect to the convection time of the structure across the measuring probe) and

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_x = -V \left.\frac{\partial u'}{\partial \xi}\right|_\tau$$

making use of the identity

$$\left.\frac{\partial}{\partial \xi}\right|_\tau = \left.\frac{\partial}{\partial x}\right|_t$$

and assuming the convection velocity to be equal to mean velocity U, then

$$\left.\frac{\partial u}{\partial t}\right|_x = -\bar{U} \left.\frac{\partial u}{\partial x}\right|_t$$

The above is valid, as stated, if the convection velocity is steady and if the turbulence patterns are frozen over a given time scale. If this can be accpected for fine scale turbulence, experimental results from spectral analysis and space time correlation show that it cannot be accepted for the larger flow structures. Hence a new criterion is required to correct the results for the effects caused by deviation from Taylor hypothesis. Lumley (Ref. 16) showed that this could be achieved by introducing the concept of a spatially uniform fluctuating convection velocity. That is, smaller scale turbulence is considered as frozen and convected by a time dependent velocity with characteristics similar to those of the energy-containing eddies, so that

$$V = \bar{V} + v'$$

with, as an order of magnitude

$$\overline{v'^2} = \overline{u'^2}$$

The original development leads to relations between measured and true one dimendional spectra for the fluctuating longitudinal velocity component. From this relations could be obtained for correcting the various velocity derivative statistics. (It should be noted that only statistic values could now be corrected because the convection velocity itself is fluctuating). For instance :

$$\overline{\left(\frac{\partial u}{\partial t}\right)^2} = \overline{\left(\frac{\partial u}{\partial x}\right)^2}\ \overline{U^2}\left(1 + \frac{\overline{u_1^2} + 2(\overline{u_2^2} + \overline{u_3^2})}{\overline{U^2}}\right)$$

to be compared with the above expression from Taylor assumption for turbulent intensities of the order of 30% , the difference is of the order of 50%.

This type of analysis is necessary for the interpretation of the data for spectral analysis of the flow field but it is also important for the study of other turbulence properties such as probability density distribution of velocity derivatives quantities used, for instance as detectors in conditional sampling. Furthermore, it may threw some light on the interpretation of results obtained in the analysis of the coherent structure of turbulence and in the role of phase jitter (similar to the effect of unsteady convection velocity) on the results obtained.

Author	Ref.	k, ρ, η at T	Correlation equation	Re
King	3			
Hilpert	5	T_∞	$Nu = 0.318 + 0.690\, Re^{0.5}$	$0.055 < Re < 55$
Kramers	43		$Nu = 0.891\, [Re(T_\infty/T_f)^{0.25}]^{0.330}$	$1 < Re < 4$
Ulsamer	41	T_∞	$Nu = 0.039 + 0.51\, Re^{0.5}$	$0.01 < Re < 10^4$
Eckert and Soehngen	44		$Nu = 0.43 + 0.48\, Re^{0.5}$	
McAdams	42	T_∞ but T_f for ρ	$Nu = 0.32 + 0.43\, Re^{0.52}$	$1 < Re < 4 \times 10^3$
Scadron and Warshawsky	45	T_f	$Nu = 0.431\, Re^{0.5}$	$0.1 < Re < 10^3$
Churchill and Brier	46	T_f	$Nu = 0.535\, Re^{0.5}\, (T_f/T_\infty)^{0.12}$	$250 < Re < 3 \times 10^4$
Douglas and Churchill	47	T_∞	$Nu = 0.46\, Re^{0.5} + 0.00128\, Re$	$300 < Re < 2300$
Van Der Hegge Zijnen	12	T_∞	$Nu = 0.35 + 0.5\, Re^{0.5} + 0.001\, Re$	$500 < Re < 3 \times 10^5$
Glawe and Johnson	48	T_f	$Nu = 0.428\, Re^{0.5}$	$0.01 < Re < 10^4$
Collis and Williams	16	T_∞	$Nu\, (T_\infty/T_f)^{-0.17} = 0.24 + 0.56\, Re^{0.45}$	$400 < Re < 3 \times 10^3$
Baldwin *et al.*	49			$0.02 < Re < 44$
Laurence and Sandborn	50	T_f	Graphical correlation	$0.02 < Re < 3 \times 10^5$
Davies and Fisher	18	T_f	$Nu = (2$ [illegible]$)\, Re^{0.33}$	$(0.001 < Kn < 37)$
Richardson	51		$Nu = 0.3737 + 0.37\, Re^{0.5} + 0.056\, Re^{0.66}$	$0 < Re < 50$
Parnas	20	T_f	$Nu = 0.823\, Re^{0.5}\, (T_\infty/T_f)^{[illegible]}$	$1 < Re < 10^5$
Ahmad	30	T_∞	$Nu\, (\nu_f/\nu_\infty)^{0.15} = 0.2068 + 0.4966\, Re^{0.45}$	$10 < Re < 60$
Present		T_∞	$N_e = 0.34 + 0.65\, Re^{0.45}$	$4 < Re < 40$
				$0.015 < Re < 20$; $l/d > 400$

2

TABLE 1 - HOT WIRE CORRELATIONS FOR HEAT TRANSFER

(from Ref. 2)

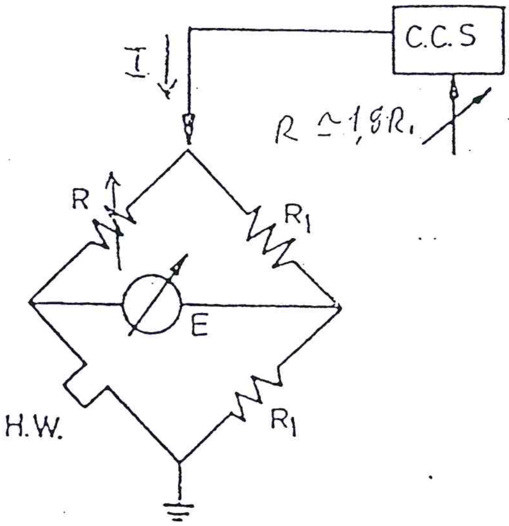


FIG. 4

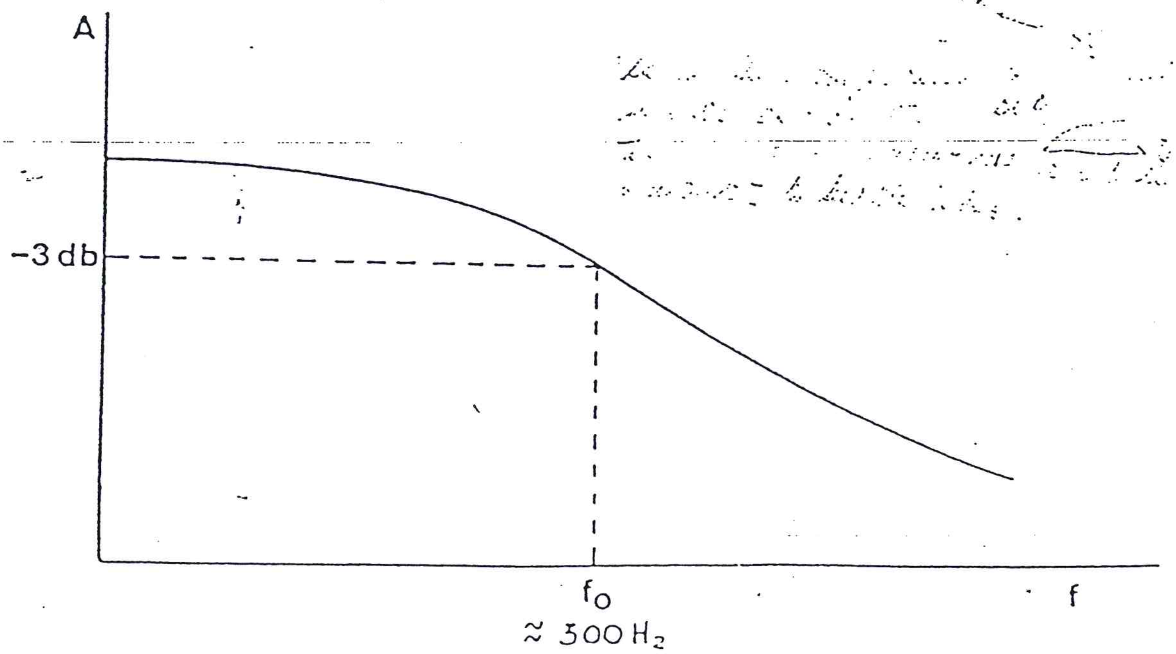


FIG. 5

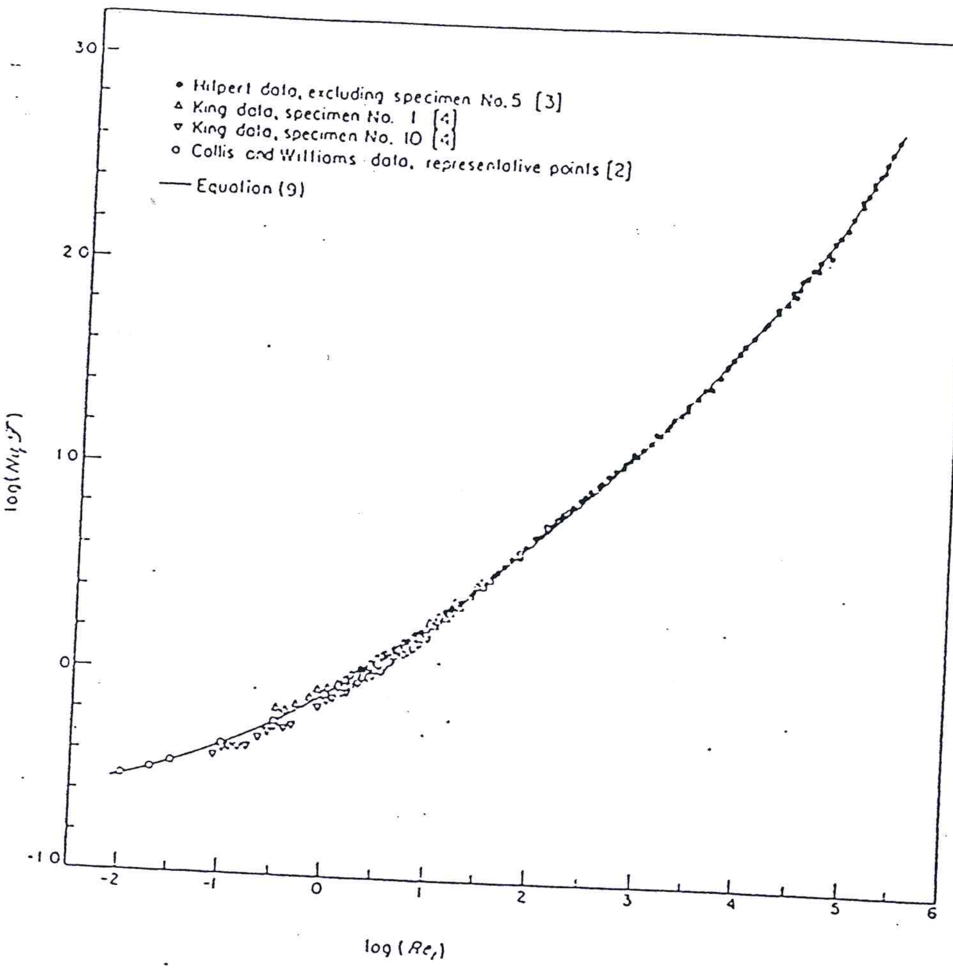


FIG. 3 - CORRELATIONS FOR HEAT TRANSFER
(from Ref. 17)

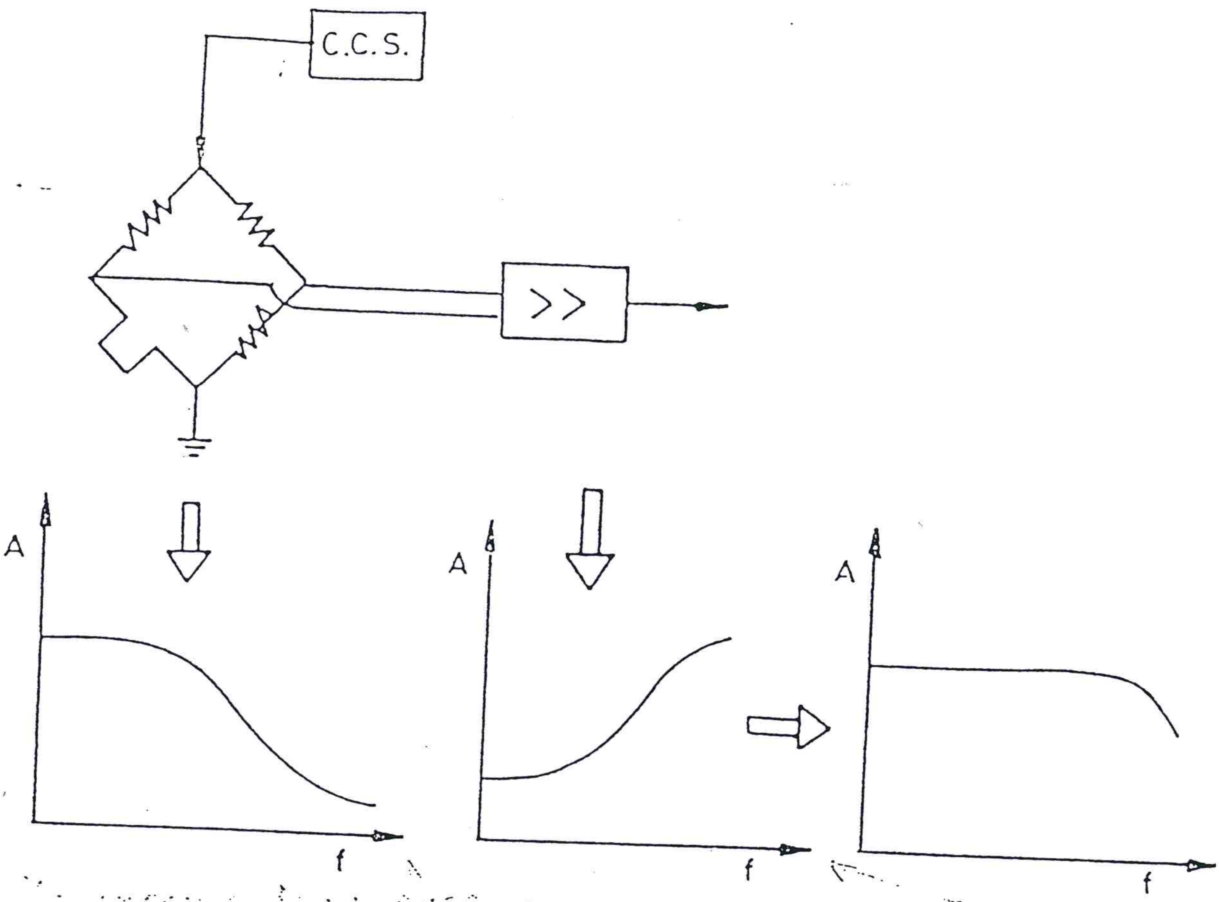
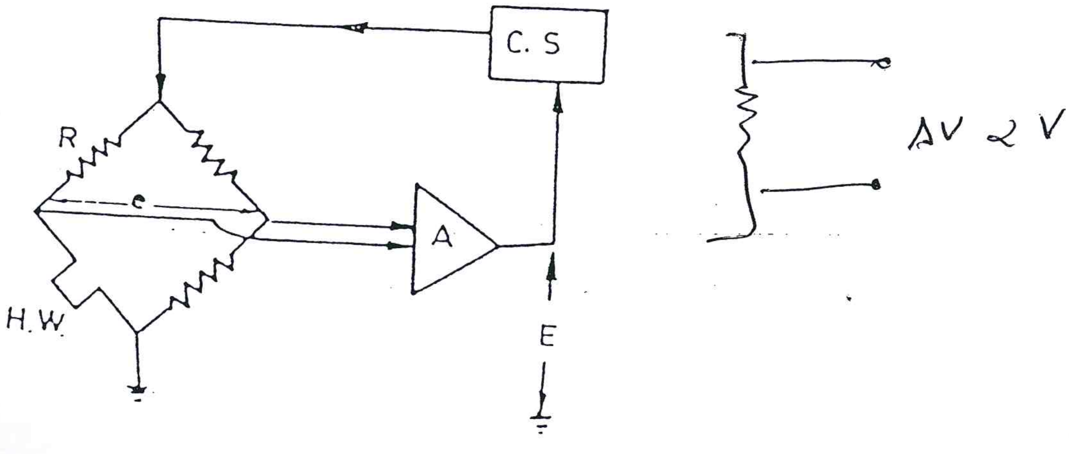


FIG. 6



A TEMPERATURA COSTANTE

FIG. 7

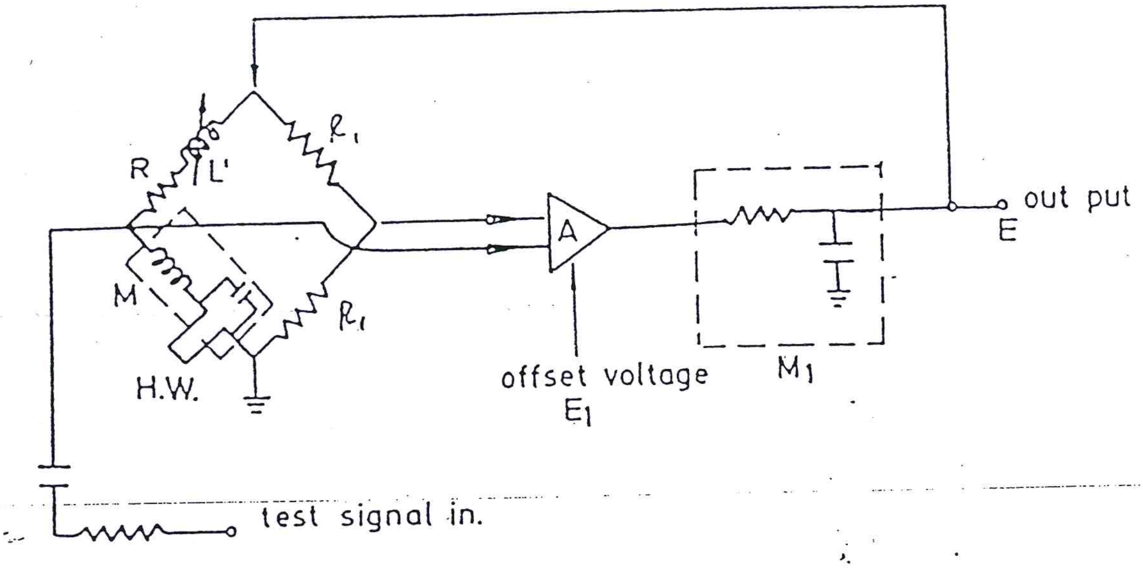


FIG. 8

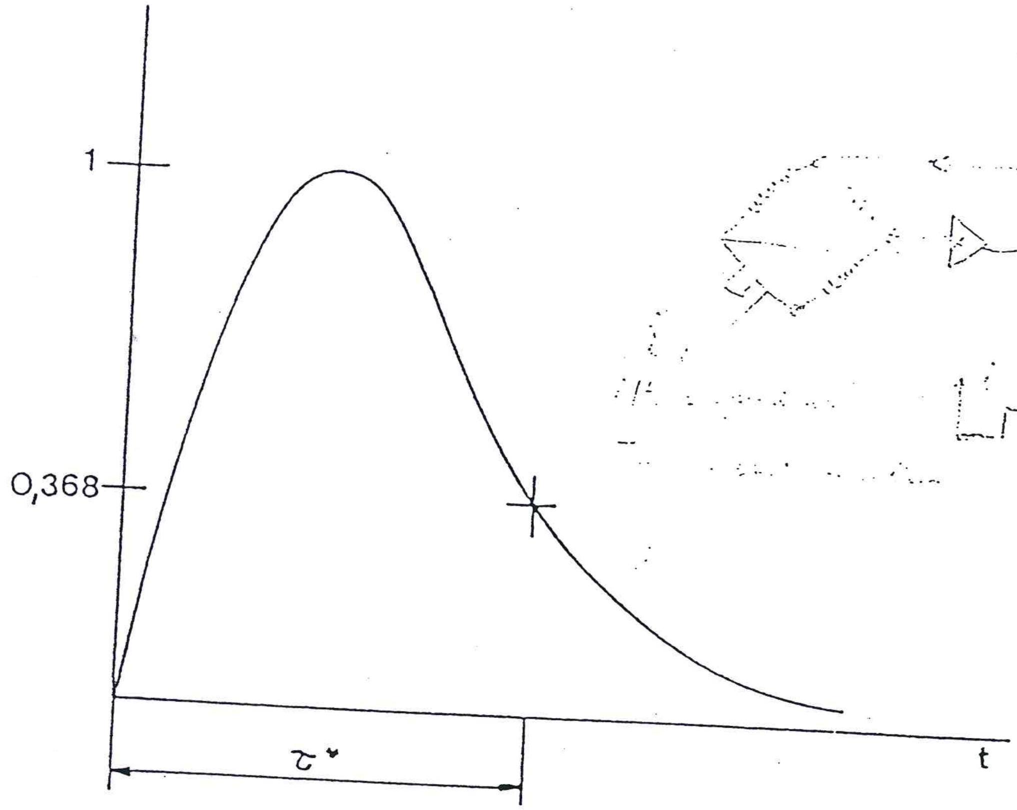


FIG. 9

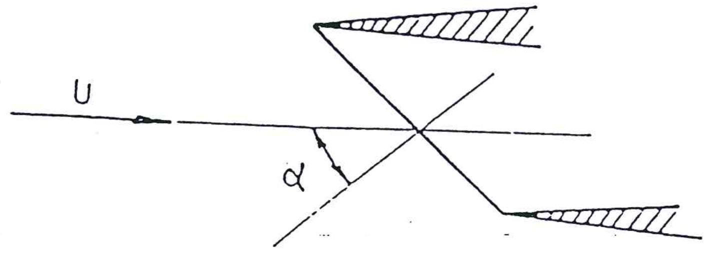
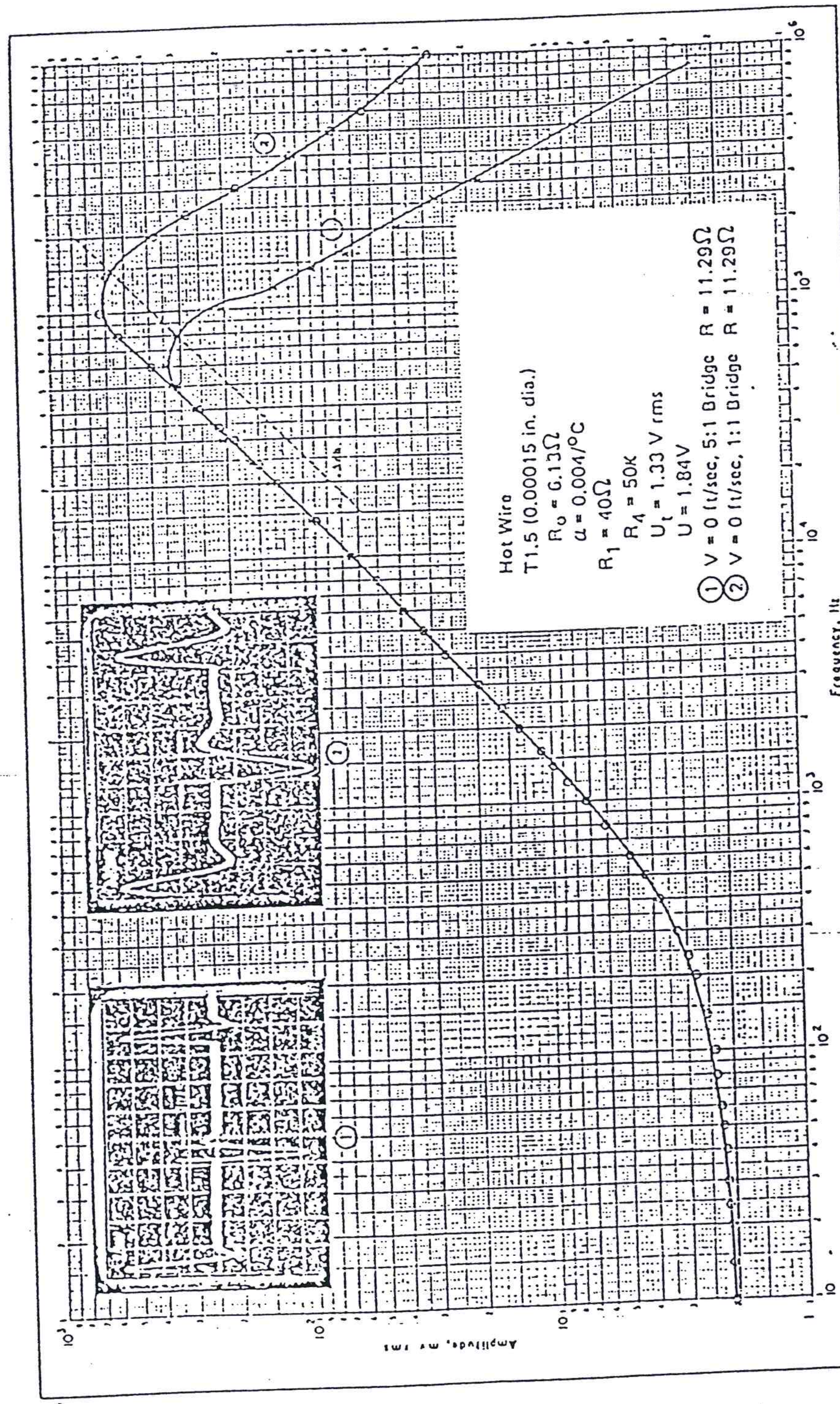
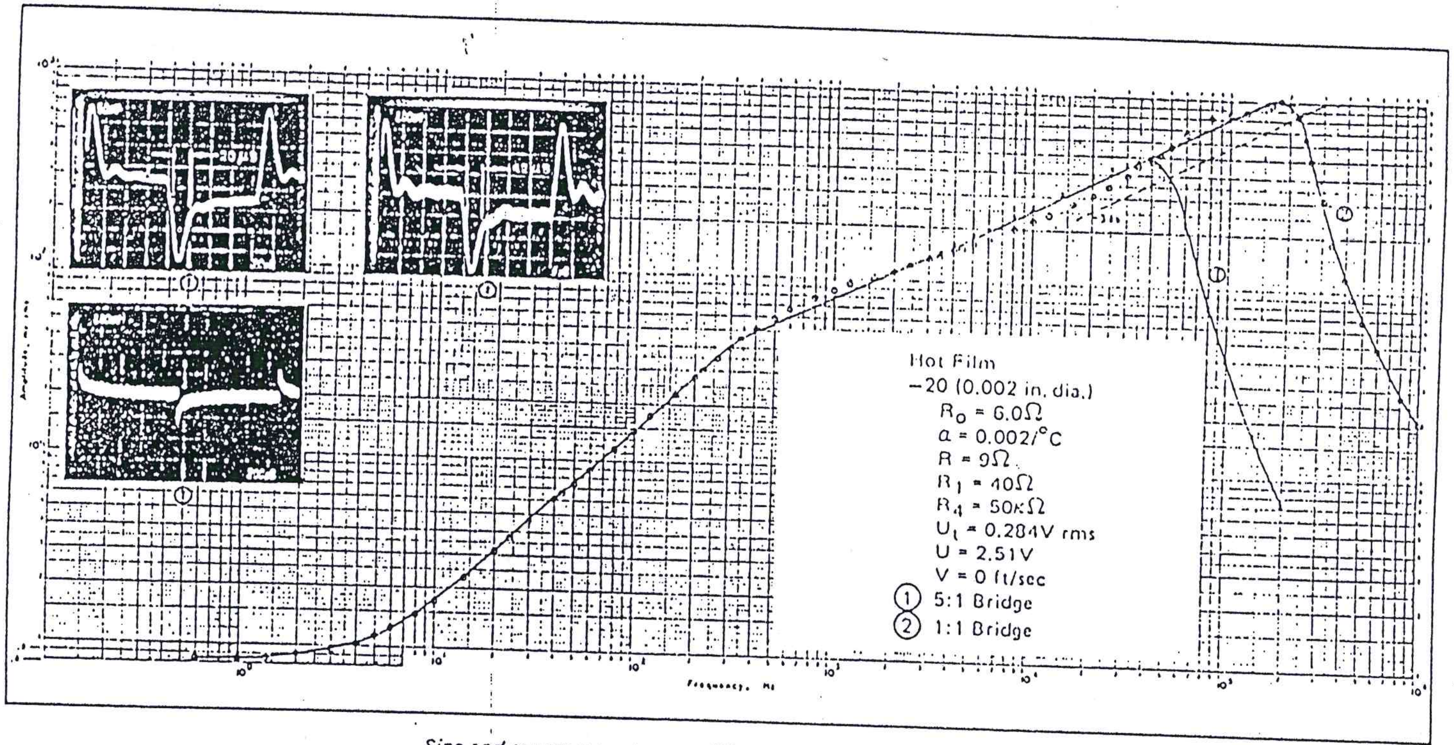


FIG. 12



Sine and square wave tests on TSI T1.5 Hot Wire at zero flow in air.

FIG. 10 (from Ref. 6)



Sine and square wave tests on TSI-20 Hot Film Sensor with no flow.

FIG. 11 (from Ref. 6)

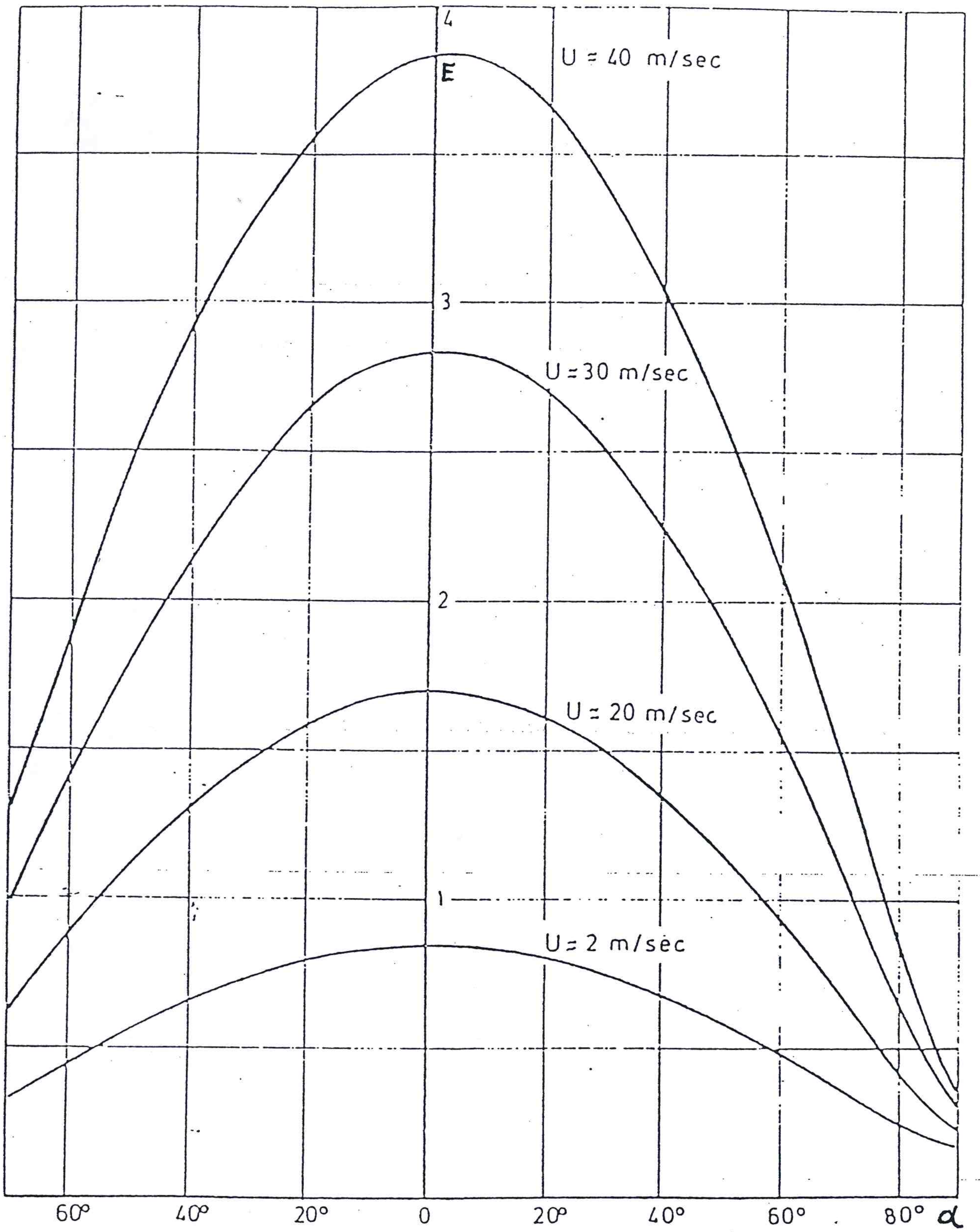


FIG. 13

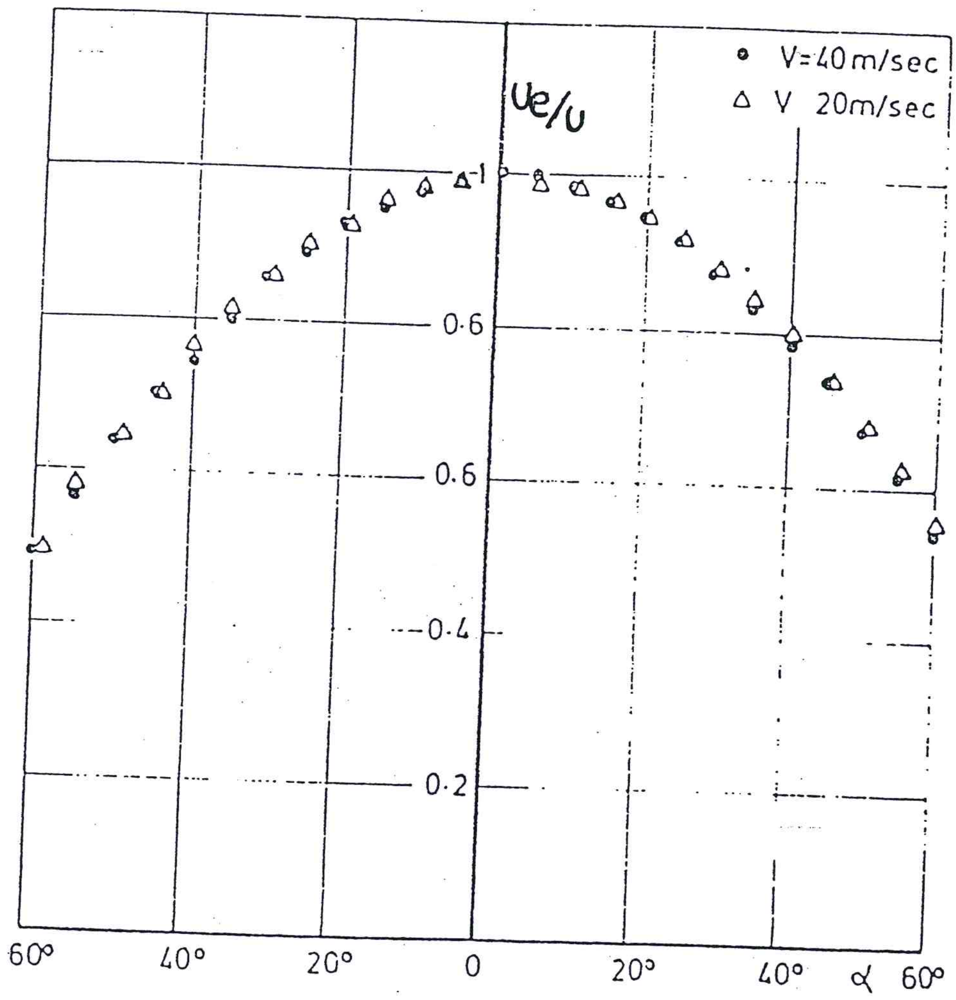


FIG. 14

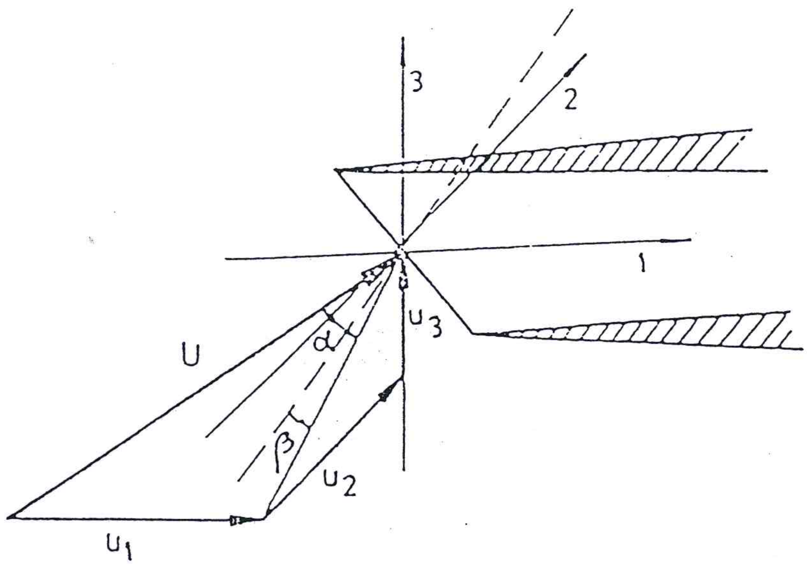


FIG. 15

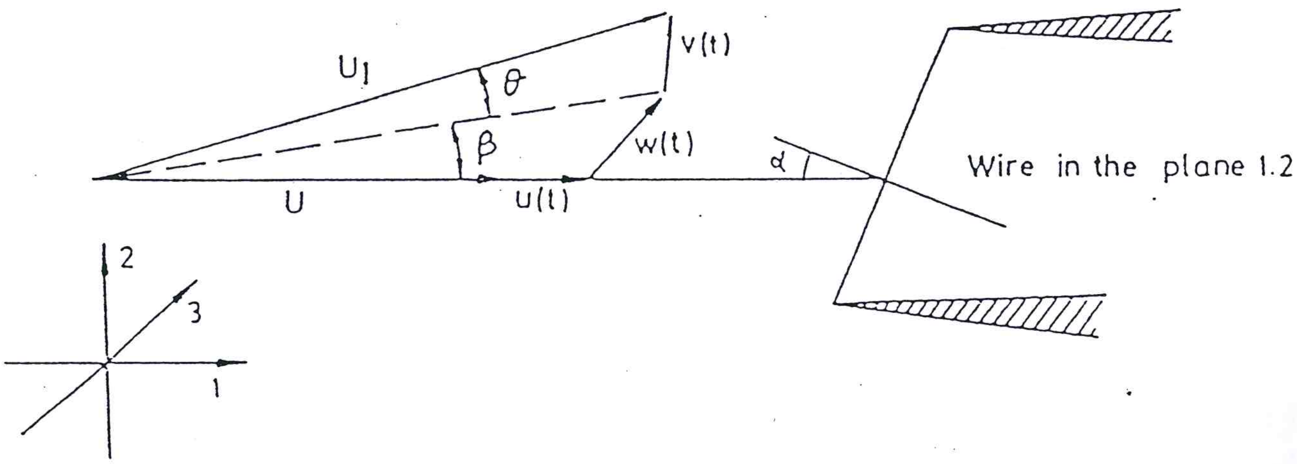


FIG. 16

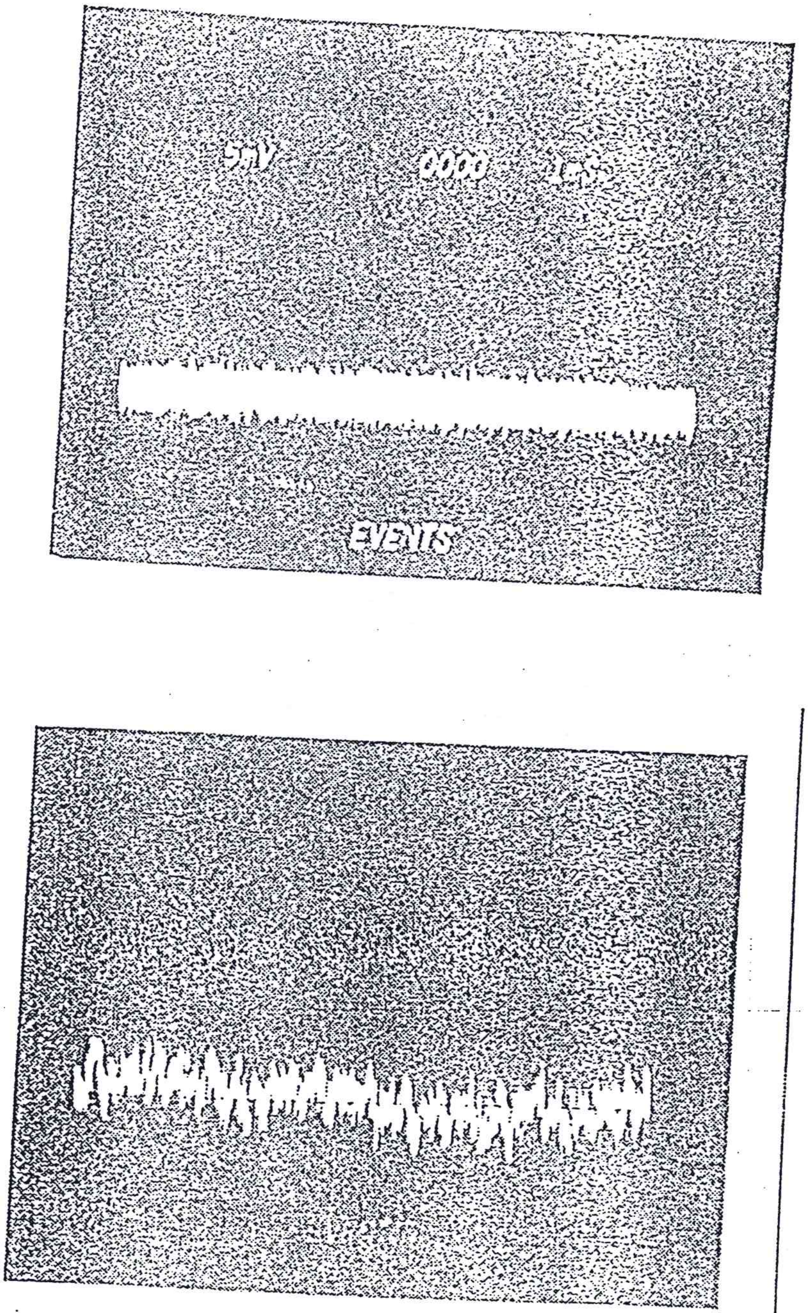


FIG. 17 - NOISE AT C.T. ANEMOMETER OUTPUT

Upper U = 0 m/sec (V = 1.49 Volt)

Lower U = 15 m/sec (V = 2.49 Volt)

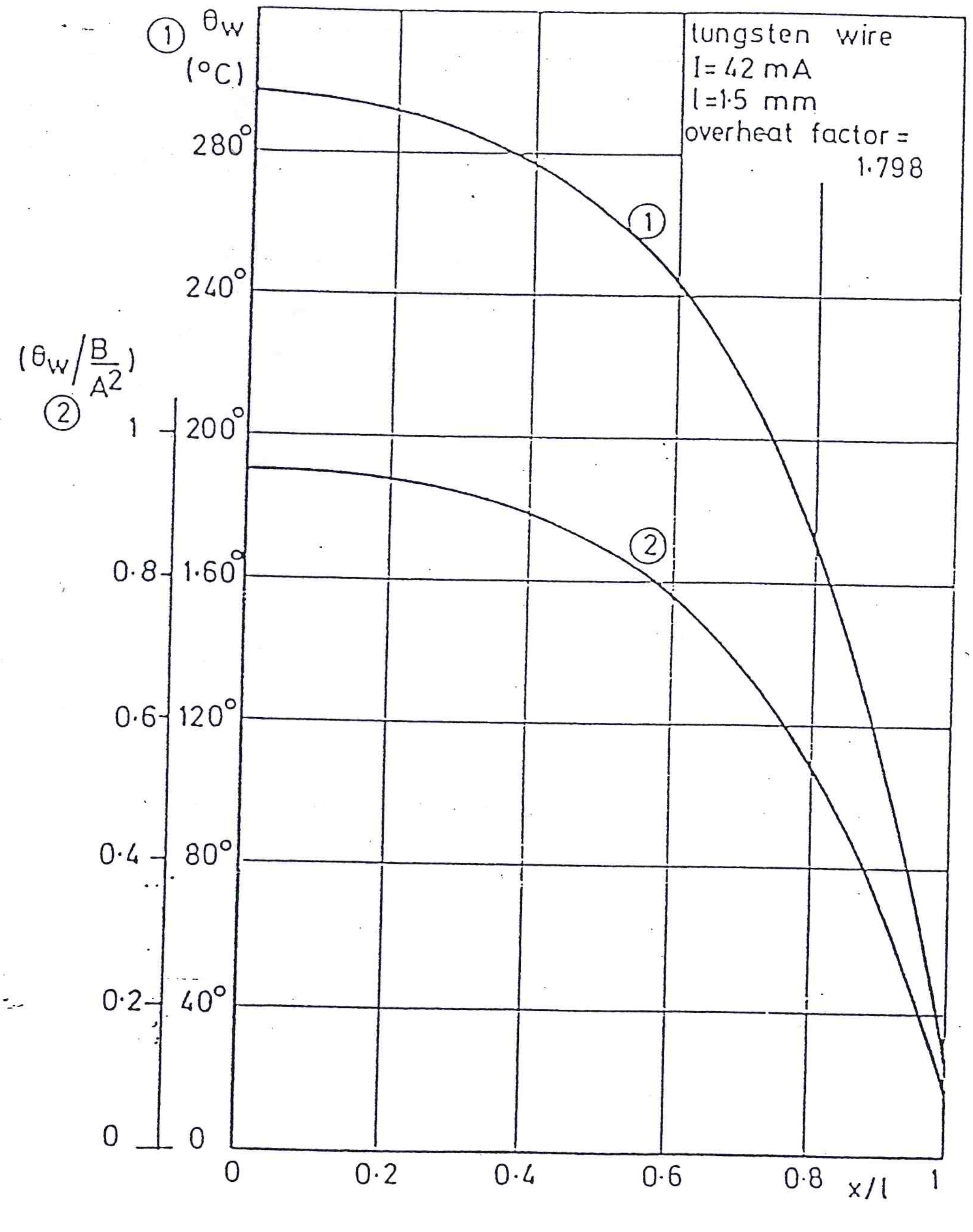
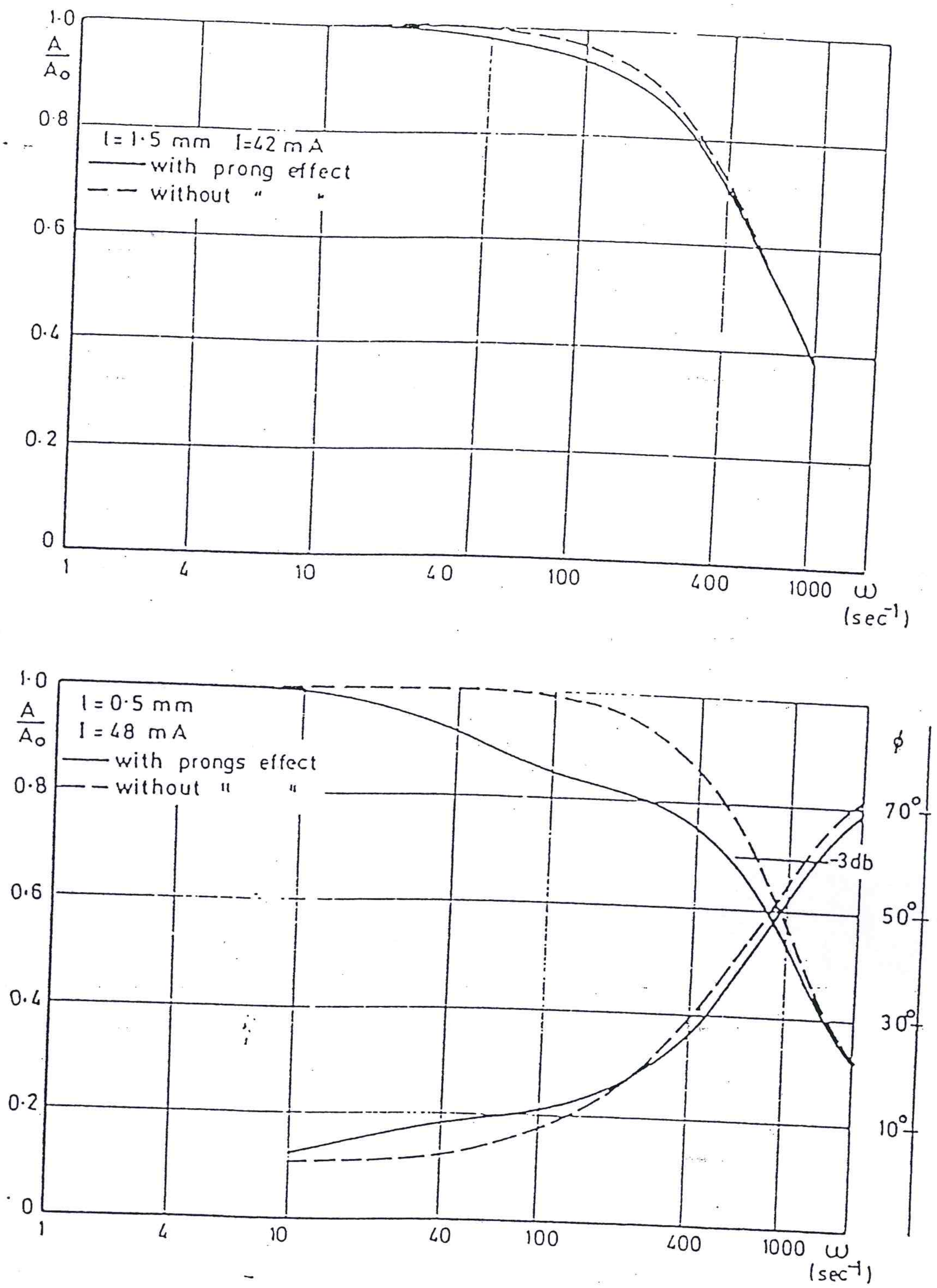
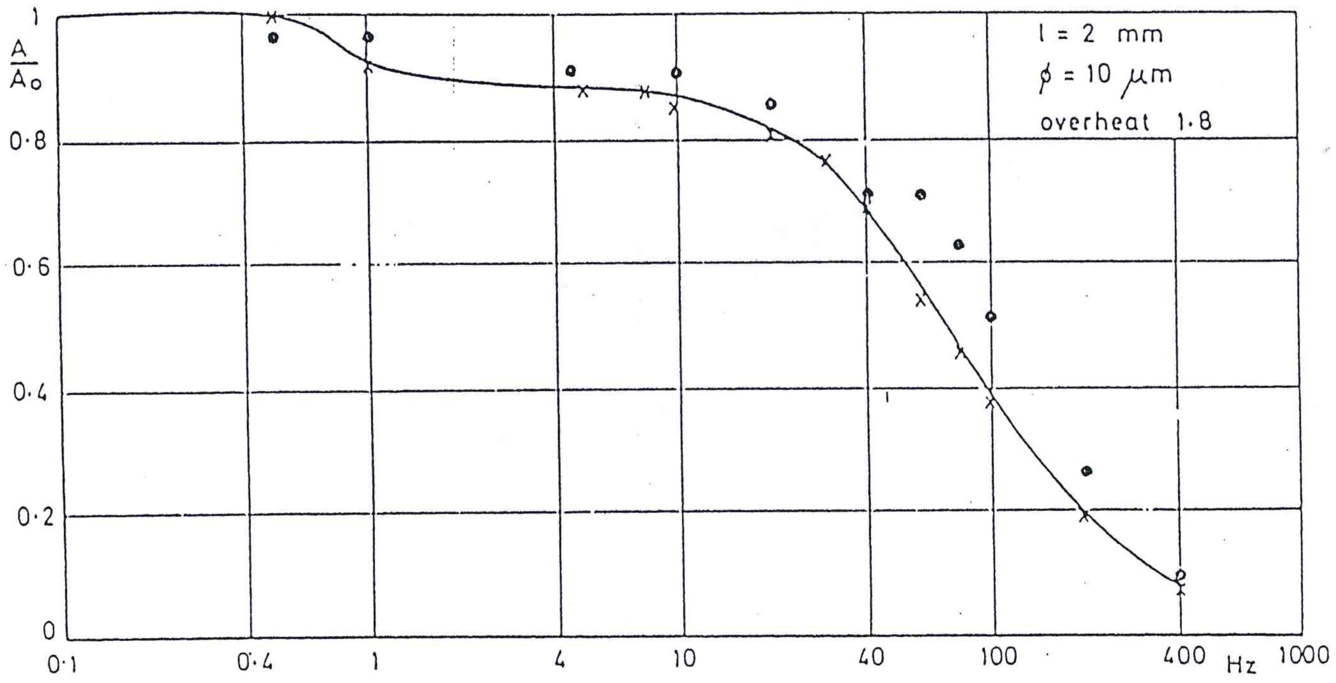


FIG. 18 - TEMPERATURE DISTRIBUTION ALONG THE WIRE
(from Ref. 10)



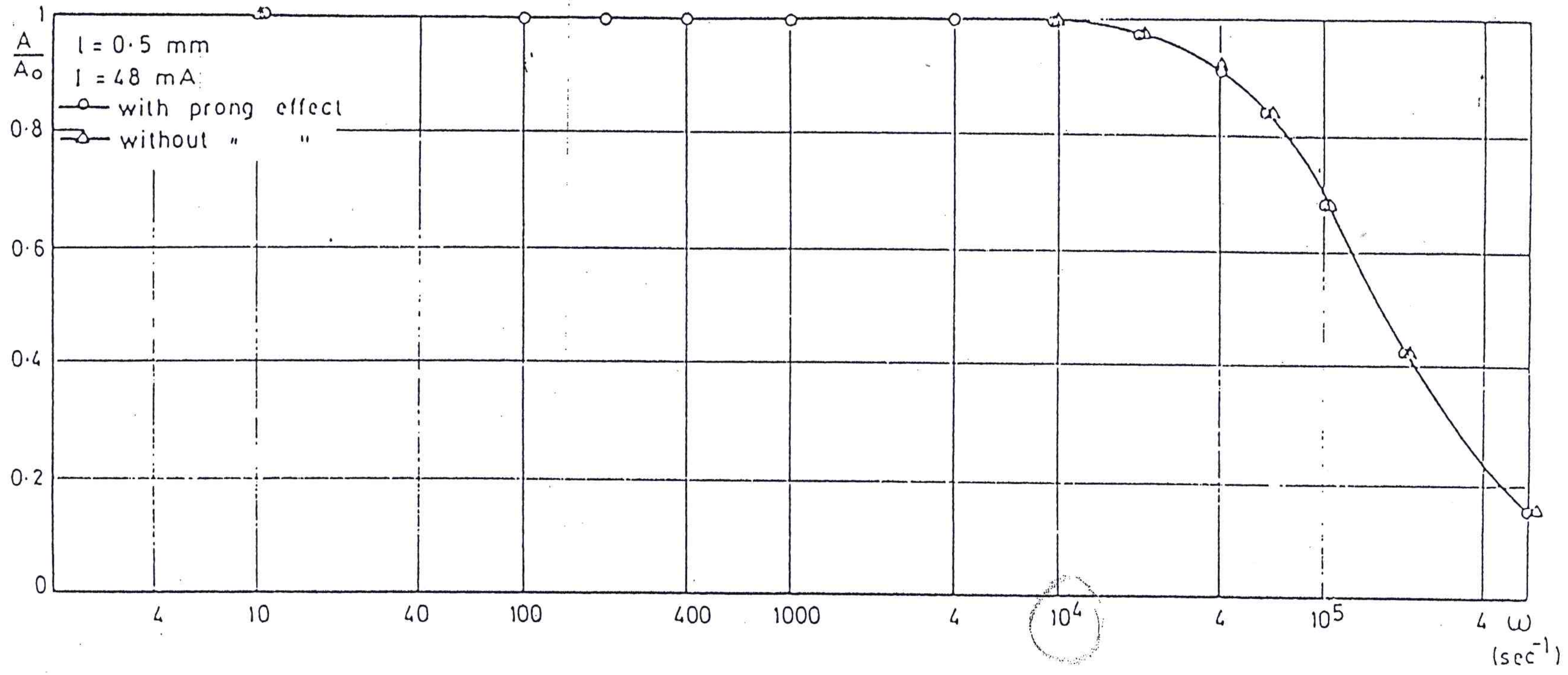
FREQUENCY RESPONSE - CONSTANT CURRENT ANEMOMETER

FIG. 19 (from Ref. 10)



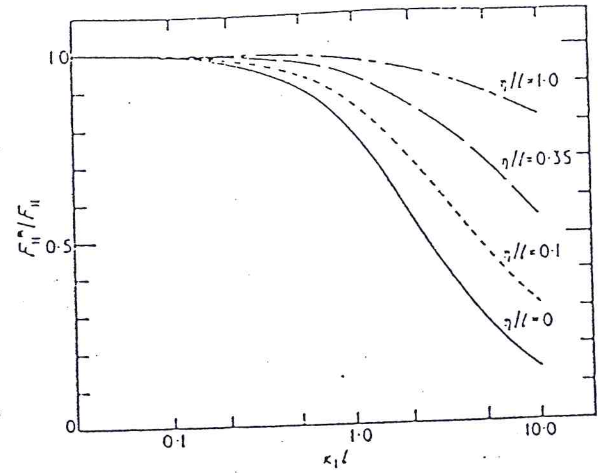
FREQUENCY RESPONSE - EXPERIMENTAL - CONSTANT CURRENT ANEMOMETER

FIG. 19 bis (from Ref. 10)

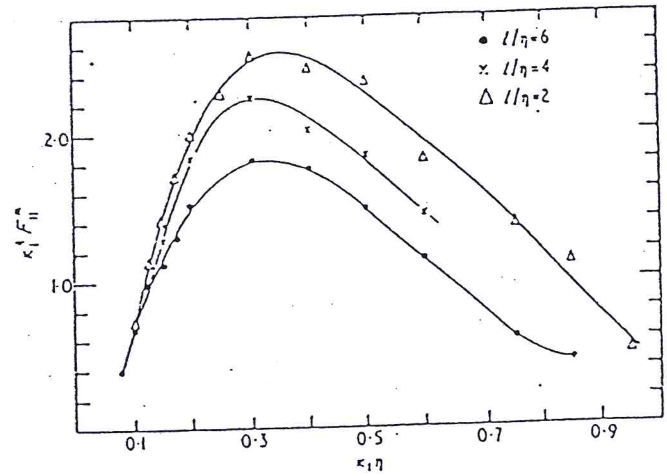


FREQUENCY RESPONSE - CONSTANT TEMPERATURE ANEMOMETER

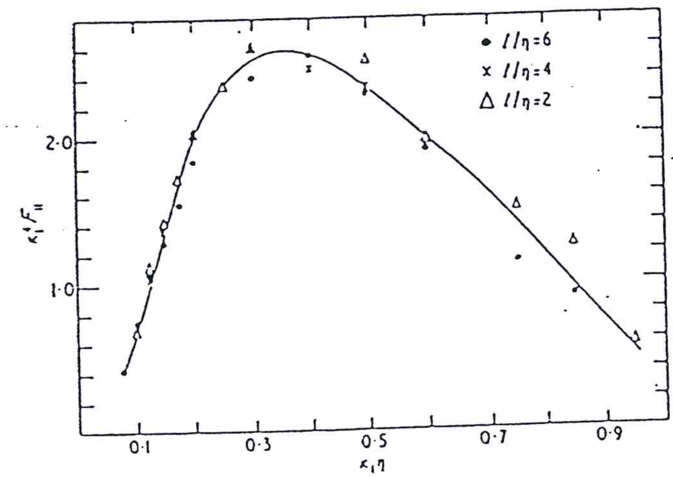
FIG. 20 (from Ref. 10)



One-dimensional spectral response for single wire

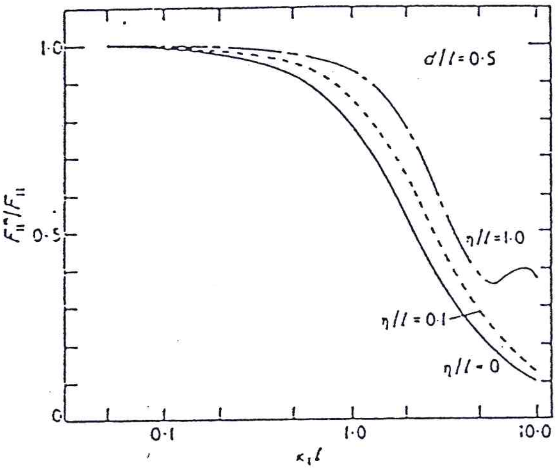


Measured one-dimensional spectra

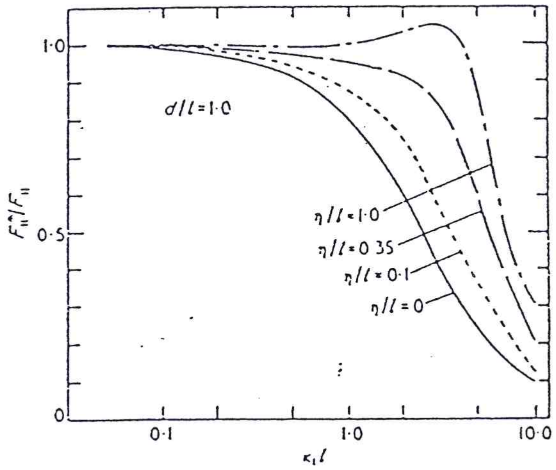


Corrected one-dimensional spectra

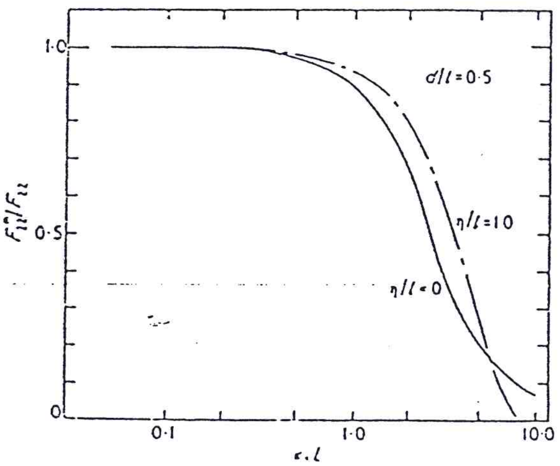
FIG. 21a - SPATIAL RESOLUTION OF HOT WIRES
(from Ref. 14)



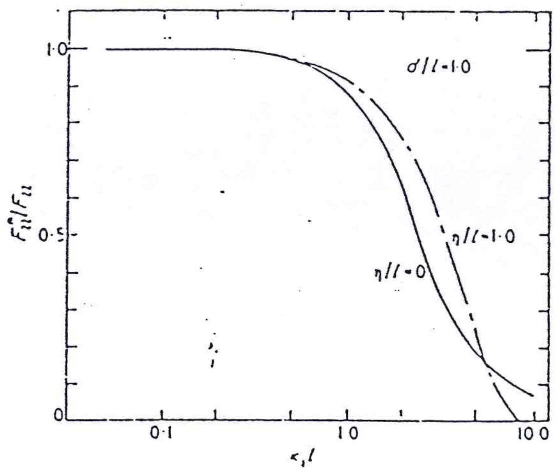
Longitudinal one-dimensional spectral response of X-array, $d/l = 0 \cdot 5$



Longitudinal one-dimensional spectral response of X-array, $d/l = 1 \cdot 0$



Lateral one-dimensional spectral response of X-array, $d/l = 0 \cdot 5$



Lateral one-dimensional spectral response of X-array, $d/l = 1 \cdot 0$

FIG. 21b - SPATIAL RESOLUTION OF HOT WIRES

(from Ref. 14)

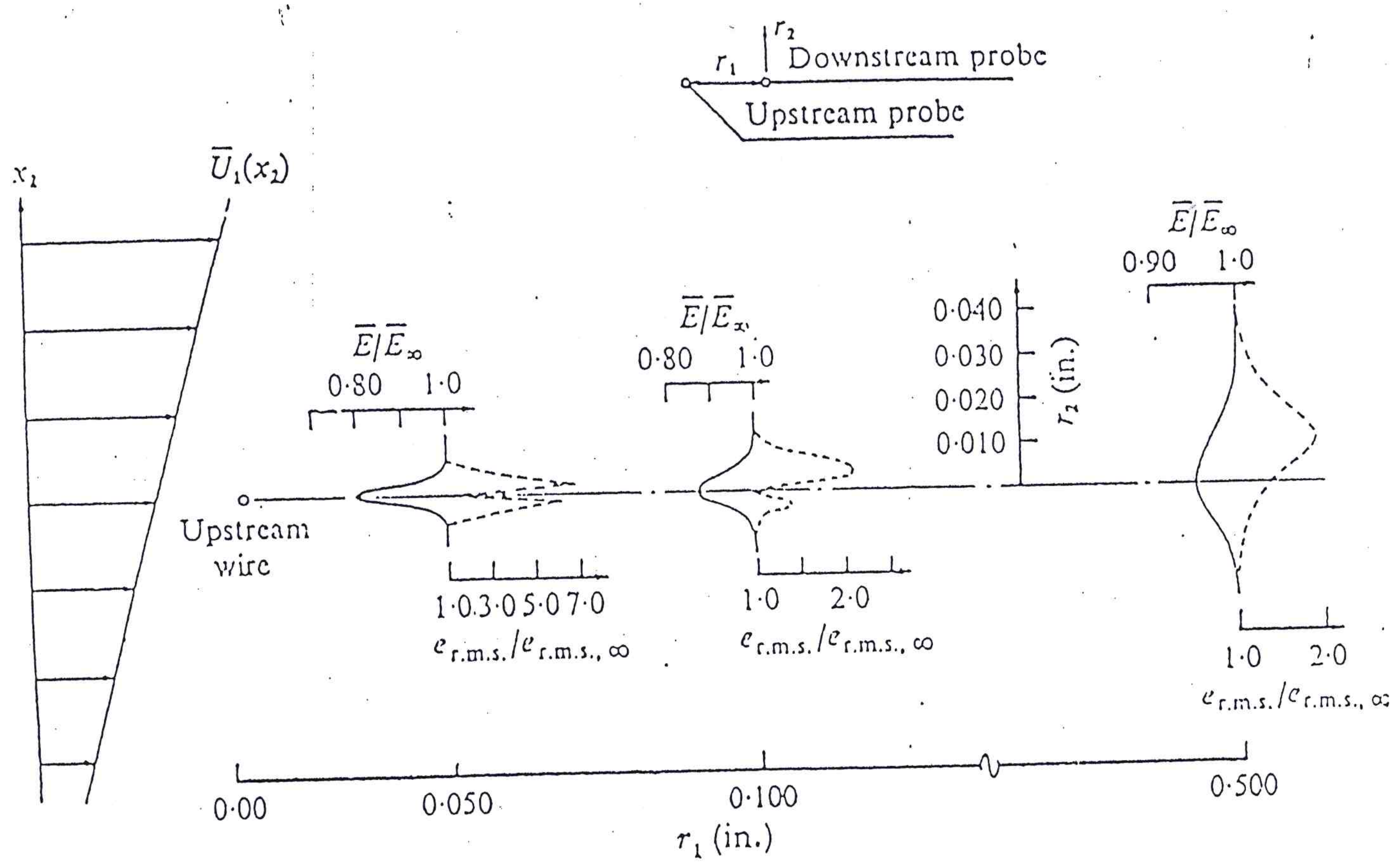


FIG. 22 - Mean velocity and fluctuation profiles close behind upstream probe.

(CHAMPAGNE, HARRIS and CORRSIN, 1970)

0. Practically all references on hot wire anemometry are listed in
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13.

14. WIJNGAARD, J.C.: Spatial averaging effects of hot wire anemometers.
a) J. Sc. Instr., Vol. 1, 1968, p. 1105
b) J. Sc. Instr., Vol. 2, 1969, p. 983
c) J. Fluid Mechanics, Vol. 48, 1971, p. 763

15. CHAMPAGNE, F.H.: The fine scale structure of the turbulent velocity field.
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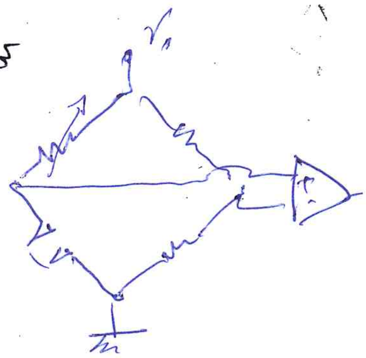
DUBBI:

- pag 3 (non compare la lunghezza del filo)
- pag 7
- pag 9
- pag 12
- pag 21 (perché non u e w)

BANDA PASSANTE A CORRENTE COSTANTE



$$I^2 \cdot R = H(V)\cdot(T_w - T_A) + C_M \frac{dT_w}{dt}$$

$$I^2 \cdot R_A \cdot [1 + b_1 \cdot (T_w - T_A)] = H(V)\cdot(T_w - T_A) + C \cdot \frac{dT_w}{dt}$$

$$\begin{cases} V \longrightarrow V_0 + v \Rightarrow H(V) \longrightarrow H(V_0) + \frac{\partial H}{\partial V} v \\ T_w \longrightarrow T_{w_0} + t_w \end{cases}$$

PER PICCOLE VARIAZIONI $\Rightarrow$

SEMPLIFICO

$$I^2 \cdot R_A [1 + b_1 (T_{w_0} - T_A)] + I^2 \cdot R_A \cdot b_1 \cdot t_w =$$

SEMPLIFICO

$$H(V)\cdot(T_{w_0} - T_A) + H(V)\cdot t_w + C \cdot \frac{dt_w}{dt} \Rightarrow$$

INF. 2° ORD.

$$I^2 \cdot R_A \cdot b_1 \cdot t_w = H(V_0)\cdot t_w + \frac{\partial H}{\partial V} \cdot v \cdot t_w + C \cdot \frac{dt_w}{dt}$$

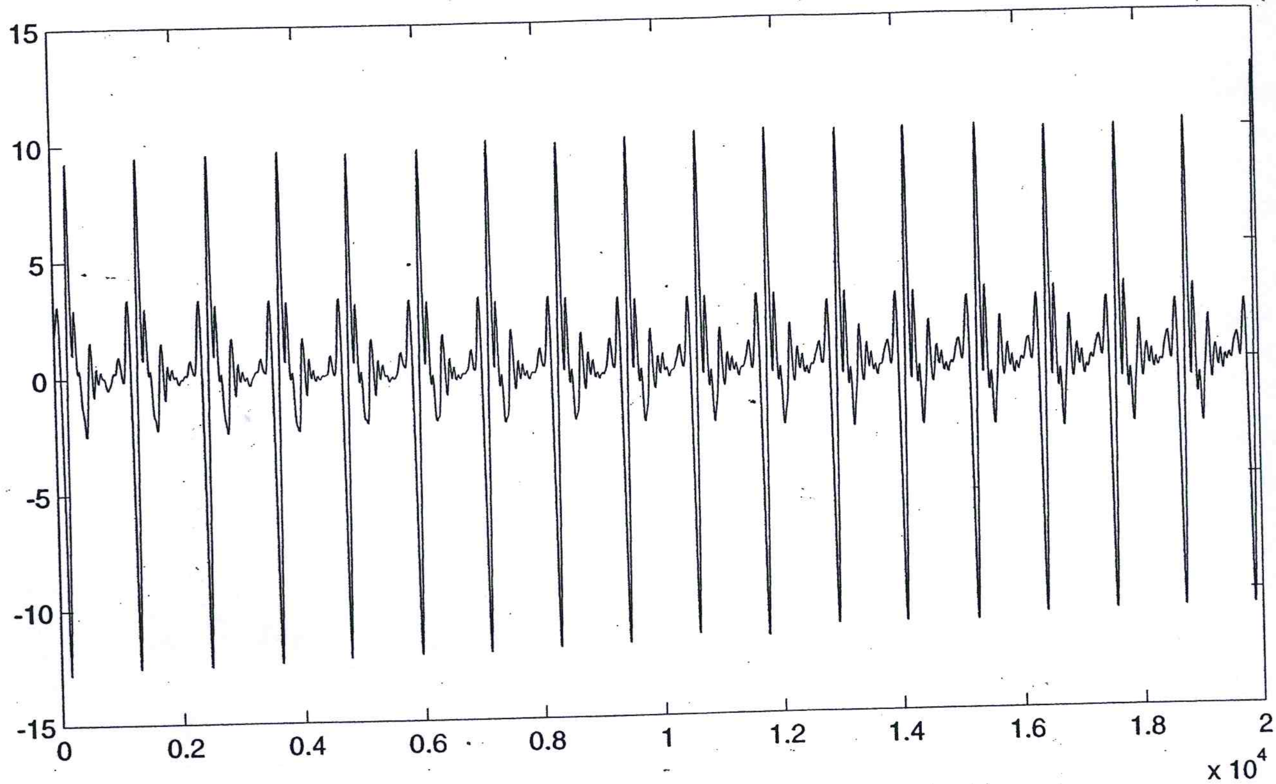
$$I^2 \cdot R_A [1 + b_1 (T_{w_0} - T_A)] + I^2 \cdot R_A b_1 t_w =$$

$$\left(H(V_0) + \frac{\partial H}{\partial V} \cdot v\right)\cdot\left(T_{w_0} - T_A + t_w\right) + C_M \cdot \frac{dt_w}{dt} =$$

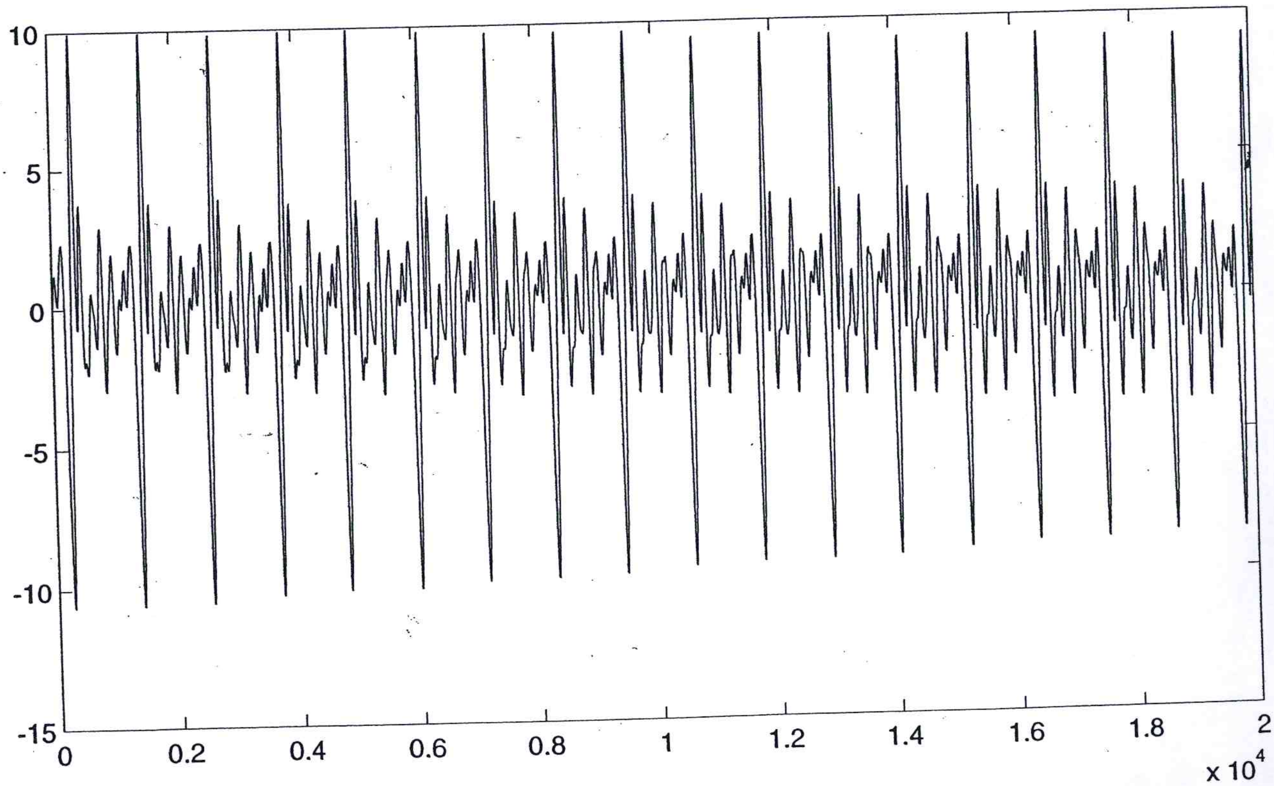
$$= H(V_0)\cdot(T_{w_0} - T_A) + H(V_0)\cdot t_w + \frac{\partial H}{\partial V} \cdot v \cdot (T_{w_0} - T_A) +$$

$$+ \frac{\partial H}{\partial V} v \cdot t_w + C_M \frac{dt_w}{dt}$$

$\ell_c = 0\%$ σ_1 ——



$\ell_c = 5\%$



$$I^2 R_A b_1 t_w = H(v_0) \cdot t_w + \frac{\partial H}{\partial v} \cdot \Delta T_0 \cdot v + c_M \frac{d t_w}{dt}$$

dunque si ottiene la funzione di trasferimento del 1° ordine:

$$c \cdot M \frac{d t_w}{dt} + \left(H(v_0) - I^2 R_A b_1 \right) \cdot t_w = -\frac{\partial H}{\partial v} \cdot \Delta T_0 \cdot v$$

↓ Trasformando secondo Fourier:

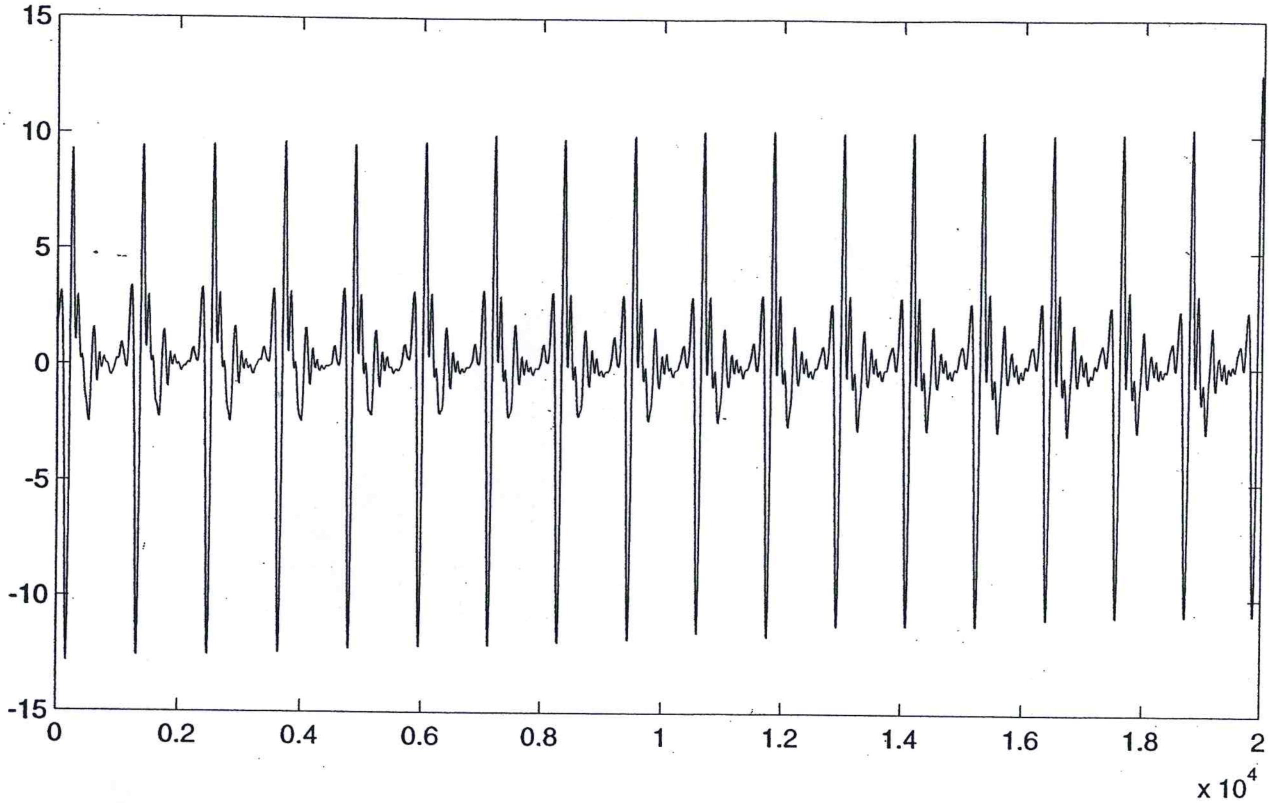
$$j\omega \cdot CM\, T_w(\omega) + \left(H(v_0) - I^2 R_A b_1 \right) \cdot T_w(\omega) = -\frac{\partial H}{\partial v} \cdot \Delta T_0 \cdot V(\omega)$$

$$\Rightarrow \frac{T_w(\omega)}{V(\omega)} = \frac{-\frac{\partial H}{\partial v} \cdot \Delta T_0}{\left(H(v_0) - I^2 R_A \right) + j\omega CM} = \frac{\dfrac{-\frac{\partial H}{\partial v} \cdot \Delta T_0}{H(v_0) - I^2 R_A b_1}}{1 + j\omega \dfrac{CM}{H(v_0) - I^2 R_A b_1}}$$

$$\Rightarrow f_{TAGLIO} = \frac{1}{\tau} = \frac{H(v_0) - I^2 R_A b_1}{CM}$$

M, C ↑ ⇒ la banda in frequenza diminuisce

I ↓ ⇒ " " " " aumenta, ma la sensibilità diminuisce l.c.=0%   db2   Δ=100   przod 34-50



l.c. =5%   db2   Δ=100