

Cosa significa misurare?

Come si misura?

Cosa bisogna scrivere per esprimere correttamente cosa si è misurato?

E' più importante la misura o l'incertezza associata?

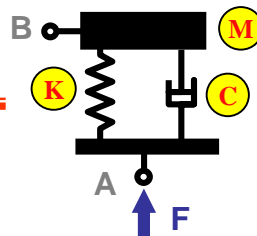
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Qualche linea guida?

ISO?



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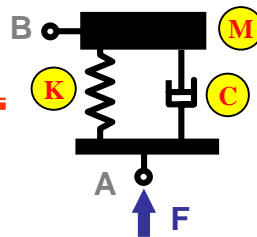
# GUM

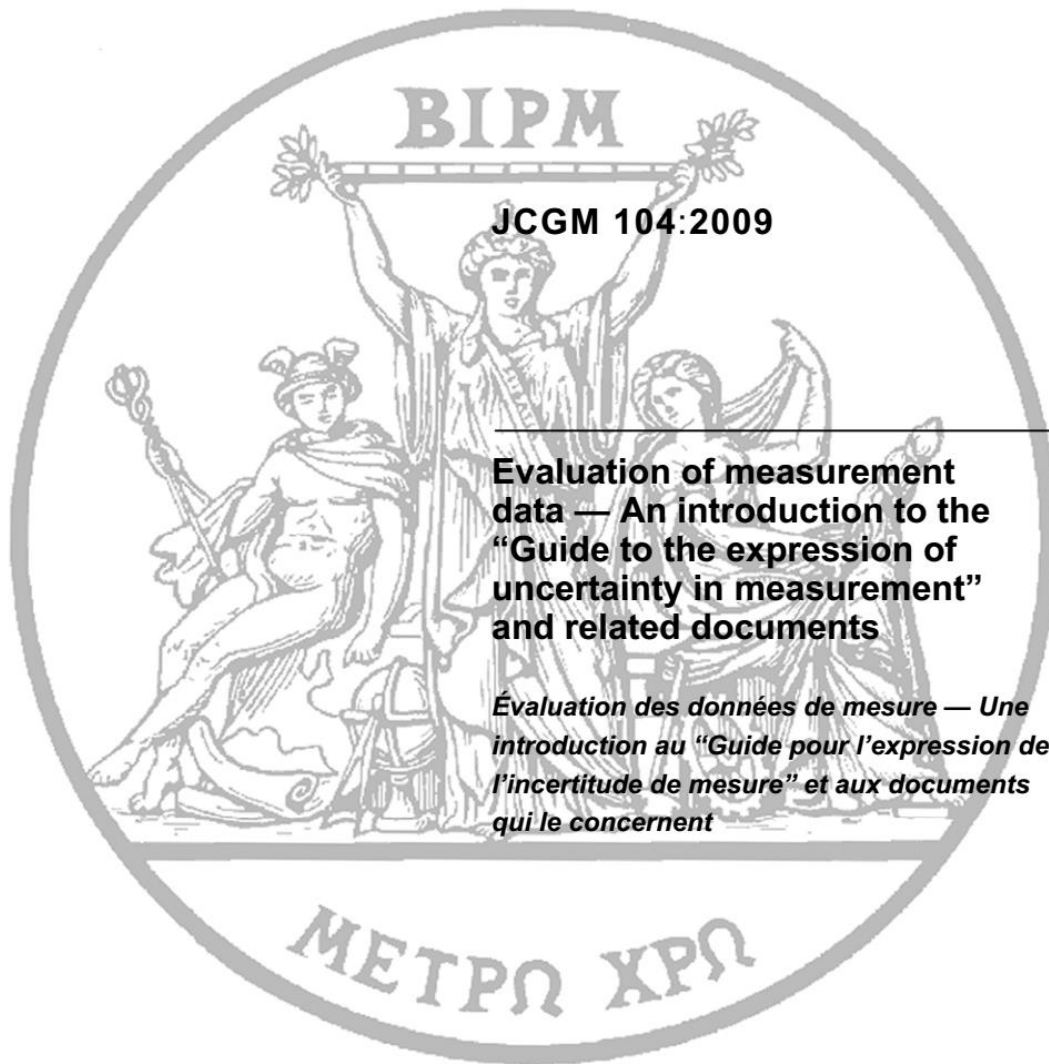
## Guide to the expression of uncertainty in measurement

<http://www.bipm.org/en/publications/guides/gum.html>



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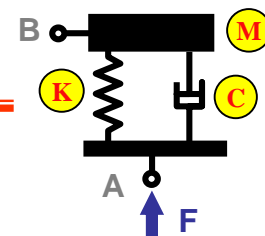
**Evaluation of measurement data — An introduction to the “Guide to the expression of uncertainty in measurement” and related documents**

*Évaluation des données de mesure — Une introduction au “Guide pour l’expression de l’incertitude de mesure” et aux documents qui le concernent*



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### 3 What is measurement uncertainty?

**3.1** The purpose of measurement is to provide information about a quantity of interest, a *measurand*.

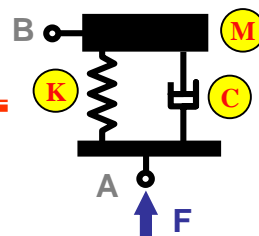
**3.2** No measurement is exact. When a quantity is measured, the outcome depends on the measuring system, the measurement procedure, the skill of the operator, the environment, and other effects. Even if the quantity were to be measured several times, in the same way and in the same circumstances, a different *indication value* would in general be obtained each time, assuming that the measuring system has sufficient resolution to distinguish between the indication values. Such indication values are regarded as instances of an indication quantity.

**3.3** The *dispersion of the indication values* would relate to how well the measurement is made. Their *average* would provide an *estimate* [ISO 3534-1:2006 1.31] of the *true quantity value* that generally would be more reliable than an individual indication value. The dispersion and the number of indication values would provide information relating to the average value as an estimate of the true quantity value. However, this information would not generally be adequate.



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**3.4** The measuring system may provide indication values that are not dispersed about the true quantity value, but about some value offset from it. The difference between the offset value and the true quantity value is sometimes called the *systematic error value*.

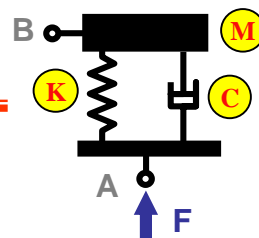
**3.5** There are **two types of measurement error quantity, systematic and random**. A **systematic error** (an estimate of which is known as a **measurement bias**) is associated with the fact that a measured quantity value contains an offset. A random error is associated with the fact that when a measurement is repeated it will generally provide a measured quantity value that is different.

**3.6** A challenge in measurement is how best to express what is learned about the measurand. Expression of systematic and random error values relating to the measurement, along with a best estimate of the measurand, is one approach that was often used prior to the introduction of the GUM. **The GUM provided a different way of thinking about measurement, in particular about how to express the perceived quality of the result of a measurement.** Rather than express the result of a measurement by providing a best estimate of the measurand, along with information about systematic and random error values (in the form of an 'error analysis'), **the GUM approach is to express the result of a measurement as a best estimate of the measurand, along with an associated measurement uncertainty.**



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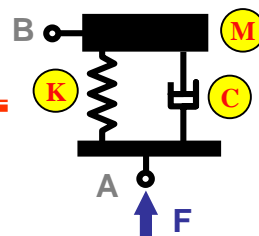
**3.7** One of the basic premises of the GUM approach is that it is possible to characterize the quality of a measurement by accounting for both systematic and random errors on a comparable footing, and a method is provided for doing that (see 7.2). This **method refines the information previously provided in an 'error analysis', and puts it on a probabilistic basis** through the concept of measurement uncertainty.

**3.8** Another basic premise of the GUM approach is that **it is not possible to state how well the essentially unique true value of the measurand is known, but only how well it is believed to be known.** Measurement uncertainty can therefore be described as a measure of how well one believes one knows the essentially unique true value of the measurand. This uncertainty reflects the incomplete knowledge of the measurand. The notion of 'belief' is an important one, since it moves metrology into a realm where results of **measurement need to be considered and quantified in terms of probabilities that express degrees of belief.**



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**3.13** As well as raw data representing measured quantity values, there is another form of data that is frequently needed in a **model**. Some such data relate to quantities representing physical constants, each of which is known imperfectly. Examples are material constants such as modulus of elasticity and specific heat. There are often other relevant data given in reference books, calibration certificates, etc., regarded as estimates of further quantities.

**3.14** The **items required by a model** to define a measurand are known as **input quantities** in a **measurement model**. The rule or model is often referred to as a **functional relationship**. **The output quantity in a measurement model is the measurand.**

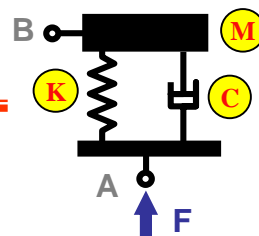
**3.15** Formally, the output quantity, denoted by  $Y$ , about which information is required, is often related to input quantities, denoted by  $X_1; \dots, X_N$ , about which information is available, by **a measurement model in the form of a measurement function**

$$Y = f(X_1, \dots, X_N).$$



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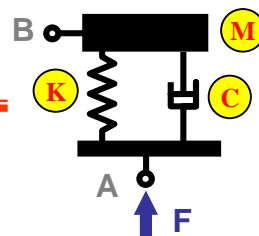
**3.18** Consider **estimates**  $x_1; \dots; x_N$ , respectively, **of the input quantities**  $X_1; \dots; X_N$ , obtained from certificates and reports, manufacturers' specifications, the analysis of measurement data, and so on. **The probability distributions characterizing**  $X_1; \dots; X_N$  are chosen such that the estimates  $x_1; \dots; x_N$ , respectively, are the **expectations** of  $X_1; \dots; X_N$ . Moreover, for the  $i$ th input quantity, consider a so-called **standard uncertainty**, given the symbol  $u(x_i)$ , defined as the **standard deviation of the input quantity  $X_i$** . This standard uncertainty is said to be *associated* with the (corresponding) estimate  $x_i$ . The estimate  $x_i$  is best in the sense that  $u^2(x_i)$  is smaller than the expected squared difference of  $X_i$  from any other value.

**3.19** The use of available knowledge to establish a probability distribution to characterize each quantity of interest applies to the  $X_i$  and also to  $Y$ . In the latter case, the characterizing probability distribution for  $Y$  is determined by the functional relationship (1) together with the probability distributions for the  $X_i$ . The determination of the **probability distribution for  $Y$  from this information is known as the propagation of distributions**.



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**4.2 Measurement uncertainty** is defined as

*non-negative parameter characterizing the **dispersion** of the quantity values being attributed to a measurand, based on the information used.*

**4.3** Two representations of a probability distribution for a random variable  $X$  are used in uncertainty evaluation:

- the **distribution function**, a function giving, for every value of its argument, the probability that  $X$  be less than or equal to that value, and
- the **probability density function**, the derivative of the distribution function.

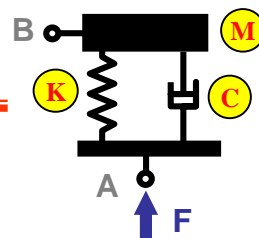
**4.4** Knowledge of each input quantity  $X_i$  in a measurement model is often summarized by the best estimate  $x_i$  and the associated standard uncertainty  $u(x_i)$ . If, for any  $i$  and  $j$ ,  $X_i$  and  $X_j$  are related (dependent), the summarizing information will also include a measure of the strength of this relationship, specified as a **covariance** or a **correlation**. If  $X_i$  and  $X_j$  are unrelated (independent), their covariance is zero.

**4.6** Knowledge about an input quantity  $X_i$  is inferred from repeated indication values (**Type A evaluation of uncertainty**), or scientific judgement or other information concerning the possible values of the quantity (**Type B evaluation of uncertainty**).



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**4.9** Once the input quantities  $X_1; \dots; X_N$  have been characterized by appropriate probability distributions, and the measurement model has been developed, the probability distribution for the measurand  $Y$  is fully specified in terms of this information. In particular, the expectation of  $Y$  is used as the estimate of  $Y$ , and the standard deviation of  $Y$  as the standard uncertainty associated with this estimate.

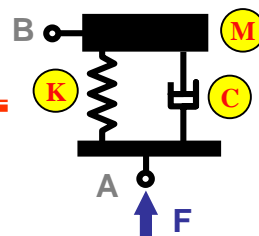
**4.14** *Sensitivity coefficients*  $c_1; \dots; c_N$  describe how the estimate  $y$  of  $Y$  would be influenced by small changes in the estimates  $x_1; \dots; x_N$  of the input quantities  $X_1; \dots; X_N$ . For the measurement function (1),  $c_i$  equals the **partial derivative of first order of  $f$  with respect to  $X_i$**  evaluated at  $X_1 = x_1; X_2 = x_2$ , etc. For the linear measurement function

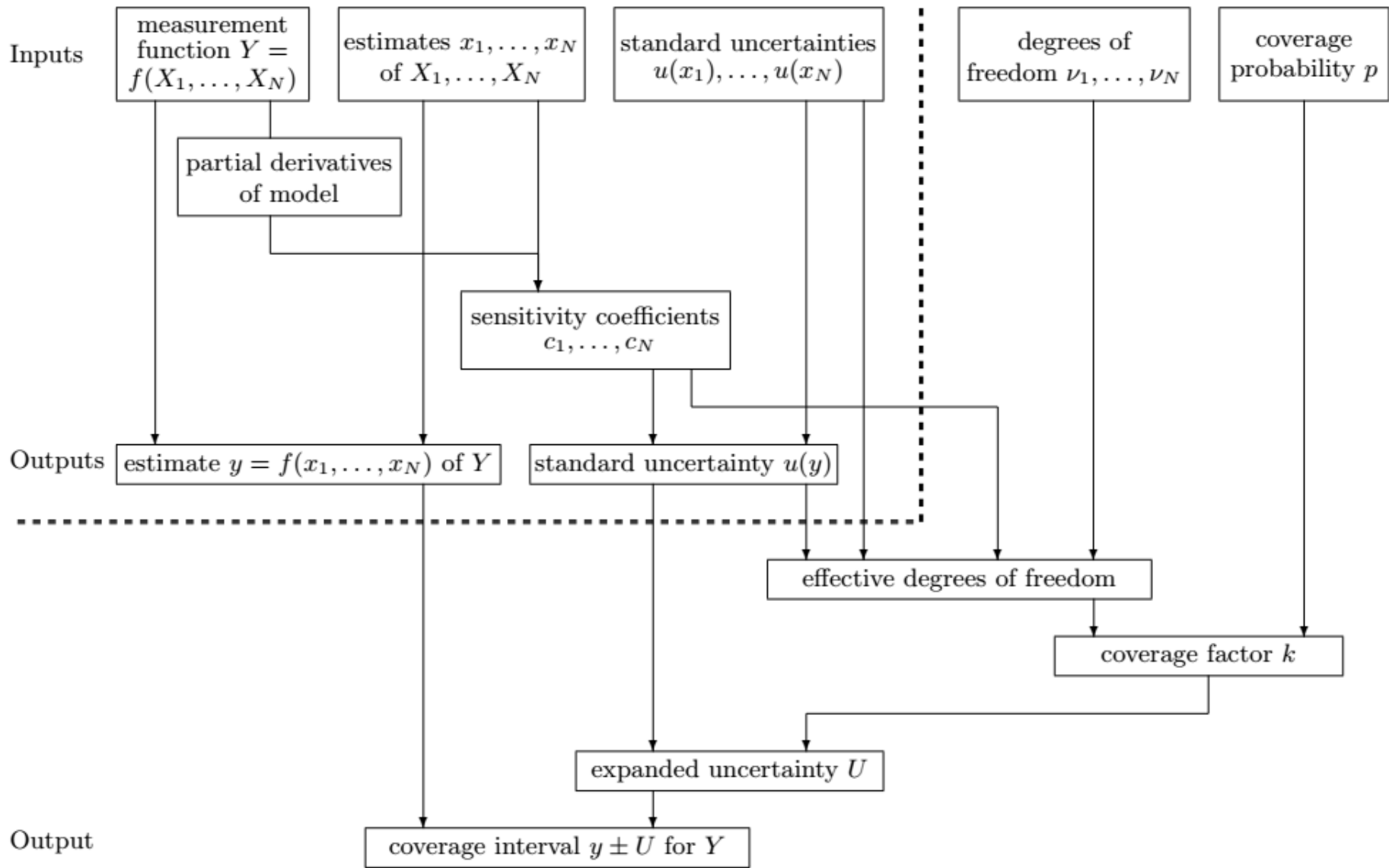
$$Y = c_1 X_1 + \dots + c_N X_N,$$

with  $X_1; \dots; X_N$  independent, a change in  $x_i$  equal to  $u(x_i)$  would give a change  $c_i u(x_i)$  in  $y$ . This statement would generally be approximate for the measurement models (1). The relative magnitudes of the terms  $|c_i|u(x_i)$  are useful in assessing the respective contributions from the input quantities to the standard uncertainty  $u(y)$  associated with  $y$ .

**4.15** The **standard uncertainty  $u(y)$  associated with the estimate  $y$**  of the output quantity  $Y$  is not given by the **sum of the  $|c_i|u(x_i)$** , but these terms combined in **quadrature**, namely by (an expression that is generally approximate for the measurement models (1) )

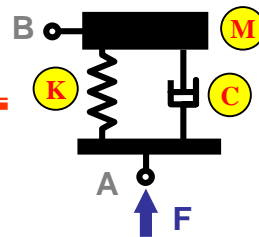
$$u^2(y) = c_1^2 u^2(x_1) + \dots + c_N^2 u^2(x_N).$$





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GUM 1995 with minor corrections

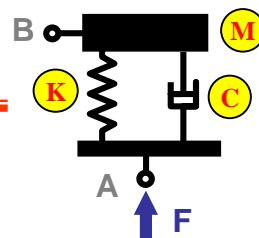
**Evaluation of measurement  
data — Guide to the expression  
of uncertainty in measurement**

*Évaluation des données de mesure —  
Guide pour l'expression de l'incertitude de  
mesure*



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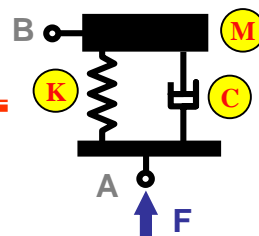
**0.1** When reporting the result of a measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared, either among themselves or with reference values given in a specification or standard. It is therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of a measurement, that is, for evaluating and expressing its *uncertainty*.

**0.2** The concept of *uncertainty* as a quantifiable attribute is relatively new in the history of measurement, although *error* and *error analysis* have long been a part of the practice of measurement science or metrology. It is now widely recognized that, when all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied, there still remains an **uncertainty about the correctness of the stated result**, that is, a doubt about how well the result of the measurement represents the value of the quantity being measured.



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**0.4** The ideal method for evaluating and expressing the uncertainty of the result of a measurement should be:

- **universal**: the method should be applicable to all kinds of measurements and to all types of input data used in measurements.

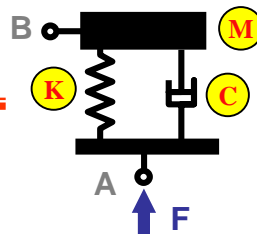
The actual quantity used to express uncertainty should be:

- **internally consistent**: it should be directly derivable from the components that contribute to it, as well as independent of how these components are grouped and of the decomposition of the components into subcomponents;
- **transferable**: it should be possible to use directly the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used.



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## 0.7 Recommendation INC-1 (1980) Expression of experimental uncertainties

- 1) The uncertainty in the result of a measurement generally consists of several components which may be grouped into two categories according to the way in which their numerical value is estimated:

- A. those which are evaluated by statistical methods,
- B. those which are evaluated by other means.

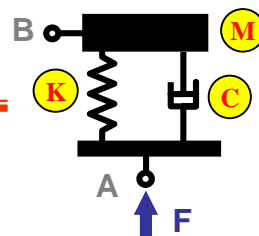
There is not always a simple correspondence between the classification into categories A or B and the previously used classification into “random” and “systematic” uncertainties. The term “systematic uncertainty” can be misleading and should be avoided. Any detailed report of the uncertainty should consist of a complete list of the components, specifying for each the method used to obtain its numerical value.

- 2) The components in category A are characterized by the estimated variances  $s_i^2$ , (or the estimated “standard deviations”  $s_i$ ) and the number of degrees of freedom  $\nu_i$ . Where appropriate, the covariances should be given.
- 3) The components in category B should be characterized by quantities  $u_j^2$ , which may be considered as approximations to the corresponding variances, the existence of which is assumed. The quantities  $u_j^2$  may be treated like variances and the quantities  $u_j$  like standard deviations. Where appropriate, the covariances should be treated in a similar way.
- 4) The combined uncertainty should be characterized by the numerical value obtained by applying the usual method for the combination of variances. The combined uncertainty and its components should be expressed in the form of “standard deviations”.
- 5) If, for particular applications, it is necessary to multiply the combined uncertainty by a factor to obtain an overall uncertainty, the multiplying factor used must always be stated.



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**2.2.1** The word “uncertainty” means doubt, and thus in its broadest sense “uncertainty of measurement” means doubt about the validity of the result of a measurement. Because of the lack of different words for **this general concept of uncertainty** and the specific quantities that provide **quantitative measures** of the concept, for example, the standard deviation, it is necessary to use the word “uncertainty” in these two different senses.

### 2.2.3 uncertainty (of measurement)

parameter, associated with the result of a measurement, that characterizes the **dispersion of the values that could reasonably be attributed to the measurand**.

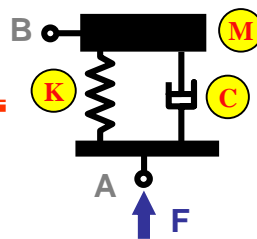
**2.2.4** The definition of uncertainty of measurement given in 2.2.3 is an operational one that focuses on the measurement result and its evaluated uncertainty. However, it is not inconsistent with other concepts of uncertainty of measurement, such as

- a measure of the possible error in the estimated value of the measurand as provided by the result of a measurement;
- an estimate characterizing the range of values within which the true value of a measurand lies



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### 2.3.1 standard uncertainty

uncertainty of the result of a measurement expressed as a **standard deviation**

### 2.3.2 Type A evaluation (of uncertainty)

method of evaluation of uncertainty by the statistical analysis of series of observations

### 2.3.3 Type B evaluation (of uncertainty)

method of evaluation of uncertainty by means other than the statistical analysis of series of observations

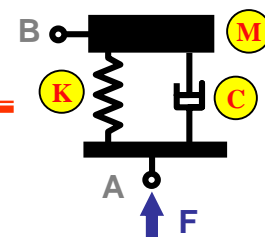
### 2.3.4 combined standard uncertainty

standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, **equal to the positive square root of a sum of terms**, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities



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## 3.1 Measurement

**3.1.1** The objective of a **measurement** is to determine the **value** of the **measurand**, that is, the value of the **particular quantity** to be measured. A measurement therefore begins with an appropriate specification of the measurand, the **method of measurement**, and the **measurement procedure**.

**3.1.3** In practice, the required specification or definition of the measurand is dictated by the required **accuracy of measurement**. The **measurand should be defined with sufficient completeness** with respect to the required accuracy so that for all practical purposes associated with the measurement its value is unique.

**3.1.4** In many cases, the result of a measurement is determined on the basis of series of observations obtained under **repeatability conditions**.

**3.1.5** Variations in repeated observations are assumed to arise because **influence quantities** that can affect the measurement result are not held completely constant

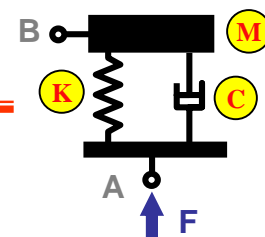
**3.1.6** The mathematical model of the measurement that transforms the set of repeated observations into the measurement result is of critical importance because, in addition to the observations, it generally includes various influence quantities that are inexactly known. **This lack of knowledge contributes to the uncertainty of the measurement result, as do the variations of the repeated observations and any uncertainty associated with the mathematical model itself.**

**3.1.7** This *Guide* treats the measurand as a scalar (a single quantity). Extension to a set of related measurands determined simultaneously in the same measurement requires replacing the scalar measurand and its **variance** by a vector measurand and **covariance matrix**.



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## 3.2 Errors, effects, and corrections

**3.2.1** In general, a measurement has imperfections that give rise to an **error** in the measurement result. Traditionally, an error is viewed as having two components, namely, a **random** component and a **systematic** component.

**3.2.2 Random error** presumably arises from **unpredictable or stochastic** temporal and spatial **variations of influence quantities**. The effects of such variations, hereafter termed *random effects*, give rise to variations in repeated observations of the measurand. Although **it is not possible to compensate for the random error of a measurement result**, it can usually be reduced by increasing the number of observations; **its expectation or expected value is zero**.

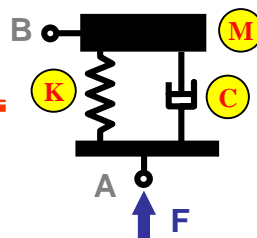
**3.2.3 Systematic error**, like random error, **cannot be eliminated but it too can often be reduced**. If a systematic error arises from a recognized effect of an influence quantity on a measurement result, hereafter termed a **systematic effect**, the effect can be quantified and, if it is significant in size relative to the required accuracy of the measurement, a **correction** or **correction factor** can be applied to compensate for the effect. It is assumed that, after correction, the expectation or expected value of the error arising from a systematic effect is zero

**3.2.4** It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects.



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## 3.3 Uncertainty

**3.3.1 The uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand.** The result of a measurement after correction for recognized systematic effects is still only an *estimate* of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects.

**3.3.2** In practice, there are many possible sources of uncertainty in a measurement, including

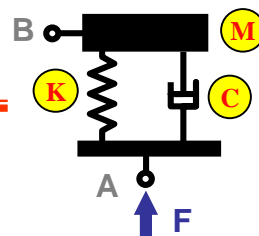
- a) incomplete definition of the measurand;
- b) imperfect realization of the definition of the measurand;
- c) nonrepresentative sampling — the sample measured may not represent the defined measurand;
- d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions;
- e) personal bias in reading analogue instruments;
- f) finite instrument resolution or discrimination threshold;
- g) inexact values of measurement standards and reference materials;
- h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm;
- i) approximations and assumptions incorporated in the measurement method and procedure;
- j) variations in repeated observations of the measurand under apparently identical conditions.

These sources are not necessarily independent, and some of sources a) to i) may contribute to source j). Of course, **an unrecognized systematic effect cannot be taken into account in the evaluation of the uncertainty of the result of a measurement but contributes to its error**



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**3.3.3** Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties groups uncertainty components into two categories based on their method of evaluation, “A” and “B”. **These categories apply to uncertainty and are not substitutes for the words “random” and “systematic”**. The uncertainty of a correction for a known systematic effect may in some cases be obtained by a Type A evaluation while in other cases by a Type B evaluation, as may the uncertainty characterizing a random effect.

**3.3.4** The purpose of the Type A and Type B classification is to indicate the two different ways of evaluating uncertainty components and is for convenience of discussion only; the classification is not meant to indicate that there is any difference in the nature of the components resulting from the two types of evaluation. **Both types of evaluation are based on probability distributions**, and the uncertainty components resulting from either type are quantified by variances or standard deviations.

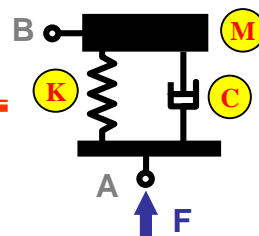
**3.3.5** The estimated variance  $u^2$  characterizing an uncertainty component obtained from a Type A evaluation is calculated from series of repeated observations and is the familiar **statistically estimated variance  $s^2$** . The estimated **standard deviation  $u$** , the positive square root of  $u^2$ , is thus  $u = s$  and for convenience is sometimes called a *Type A standard uncertainty*. For an uncertainty component obtained from a Type B evaluation, the estimated variance  $u^2$  is evaluated using available knowledge, and the estimated standard deviation  $u$  is sometimes called a *Type B standard uncertainty*.

Thus a Type A standard uncertainty is obtained from a **probability density function** derived from an **observed frequency distribution**, while a Type B standard uncertainty is obtained from an assumed probability density function based on the degree of belief that an event will occur [often called subjective **probability**]. Both approaches employ recognized interpretations of probability.



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## General metrological terms, B.2 Definitions

### B.2.1 (measurable) quantity

attribute of a phenomenon, body or substance that may be distinguished qualitatively and determined quantitatively

### B.2.2 value (of a quantity)

magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number

### B.2.3 true value (of a quantity)

value consistent with the definition of a given particular quantity

### B.2.4 conventional true value (of a quantity)

value attributed to a particular quantity and accepted, sometimes by convention, as having an uncertainty appropriate for a given purpose

### B.2.5 measurement

set of operations having the object of determining a value of a quantity

### B.2.6 principle of measurement

scientific basis of a measurement

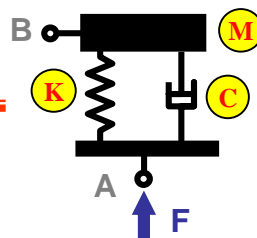
### B.2.7 method of measurement

logical sequence of operations, described generically, used in the performance of measurements



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### B.2.8 measurement procedure

set of operations, described specifically, used in the performance of particular measurements according to a given method

### B.2.9 measurand

particular quantity subject to measurement

### B.2.10 influence quantity

quantity that is not the measurand but that affects the result of the measurement

### B.2.11 result of a measurement

value attributed to a measurand, obtained by measurement

### B.2.12 uncorrected result

result of a measurement before correction for systematic error

### B.2.13 corrected result

result of a measurement after correction for systematic error

### B.2.14 accuracy of measurement

closeness of the agreement between the result of a measurement and a true value of the measurand

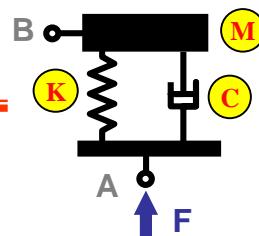
NOTE 1 “Accuracy” is a qualitative concept.

NOTE 2 The term **precision** should not be used for “accuracy”.



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### B.2.15 repeatability (of results of measurements)

closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement

### B.2.16 reproducibility (of results of measurements)

closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement

### B.2.17 experimental standard deviation

for a series of  $n$  measurements of the same measurand, the quantity  $s(q_k)$  characterizing the dispersion of the results and given by the formula:

$$s(q_k) = \sqrt{\frac{\sum_{j=1}^n (q_j - \bar{q})^2}{n-1}}$$

$q_k$  being the result of the  $k$ th measurement and  $\bar{q}$  being the arithmetic mean of the  $n$  results considered  
NOTE 1 Considering the series of  $n$  values as a sample of a distribution,  $\bar{q}$  is an unbiased estimate of the mean  $\mu_q$ , and  $s^2(q_k)$  is an unbiased estimate of the variance  $\sigma^2$ , of that distribution.

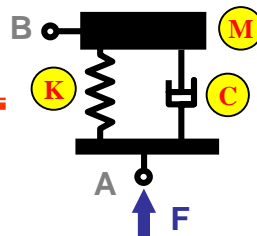
NOTE 2 The expression  $s(\bar{q})$  is an estimate of the standard deviation of the distribution of  $\bar{q}$  and is called the **experimental standard deviation of the mean**.

NOTE 3 “Experimental standard deviation of the mean” is sometimes incorrectly called **standard error of the mean**.



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### B.2.18 uncertainty (of measurement)

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

NOTE 2 Uncertainty of measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which can also be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information.

### B.2.19 error (of measurement)

result of a measurement minus a true value of the measurand

### B.2.20 relative error

error of measurement divided by a true value of the measurand

NOTE Since a true value cannot be determined, in practice a conventional true value is used

### B.2.21 random error

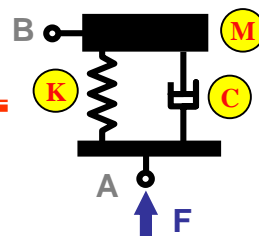
result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions

NOTE 1 Random error is equal to error minus systematic error.



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### B.2.22 systematic error

mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand

NOTE 1 Systematic error is equal to error minus random error.

NOTE 2 Like true value, systematic error and its causes cannot be completely known.

### B.2.23 correction

value added algebraically to the uncorrected result of a measurement to compensate for systematic error

NOTE 1 The correction is equal to the negative of the estimated systematic error.

NOTE 2 Since the systematic error cannot be known perfectly, the compensation cannot be complete.

### B.2.24 correction factor

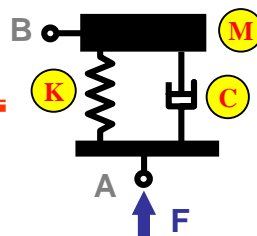
numerical factor by which the uncorrected result of a measurement is multiplied to compensate for systematic error

NOTE Since the systematic error cannot be known perfectly, the compensation cannot be complete.



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### C.2.9 expectation (of a random variable or of a probability distribution)

#### expected value, mean

For a discrete random variable  $X$  taking the values  $x_i$  with the probabilities  $p_i$ , the expectation, if it exists, is

$$\mu = E(X) = \sum p_i x_i$$

the sum being extended over all the values  $x_i$  which can be taken by  $X$ .

### C.2.10 centred random variable

a random variable the expectation of which equals zero

### C.2.11 variance (of a random variable or of a probability distribution)

the expectation of the square of the centred random variable:

$$\sigma^2 = V(X) = E\left\{[X - E(X)]^2\right\}$$

### C.2.12 standard deviation (of a random variable or of a probability distribution)

the positive square root of the variance:

$$\sigma = \sqrt{V(X)}$$

### C.2.13 central moment 2) of order $q$

in a univariate distribution, the expectation of the  $q$ th power of the centred random variable  $(X - \mu)$ :

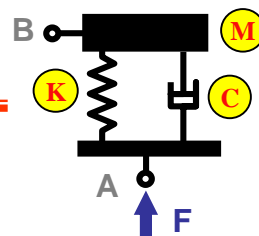
NOTE The central moment of order 2 is the variance [ISO 3534-1:1993, definition 1.22 (C.2.11)] of the random variable  $X$ .

$$E\left[(X - \mu)^q\right]$$



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### C.2.20 variance

a measure of dispersion, which is the sum of the squared deviations of **observations** from their average divided by one less than the number of observations

EXAMPLE For  $n$  observations  $x_1, x_2, \dots, x_n$  with average

$$\bar{x} = (1/n) \sum x_i$$

the variance is

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

### C.2.21 standard deviation

the positive square root of the variance

NOTE The sample standard deviation is a biased estimator of the population standard deviation.

### C.3.3 Standard deviation

The standard deviation is the positive square root of the variance. Whereas a Type A standard uncertainty is obtained by taking the square root of the statistically evaluated variance, it is often more convenient when determining a Type B standard uncertainty to evaluate a nonstatistical equivalent standard deviation first and then to obtain the equivalent variance by squaring the standard deviation.

### C.3.4 Covariance

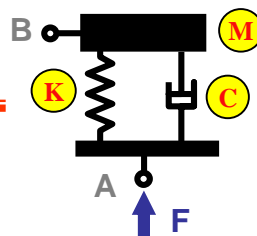
The covariance of two random variables is a measure of their mutual dependence. The covariance of random variables  $y$  and  $z$  is defined by

$$\text{COV}(y, z) = \text{COV}(z, y) = E\{[y - E(y)][z - E(z)]\}$$



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**G.1.3** To obtain the value of the **coverage factor  $k_p$**  that produces an **interval corresponding to a specified level of confidence  $p$**  requires detailed knowledge of the **probability distribution characterized by the measurement result** and its combined standard uncertainty. For example, for a quantity  $z$  described by a normal distribution with expectation  $\mu_z$  and standard deviation  $\sigma$ , the value of  $k_p$  that produces an interval  $\mu_z \pm k_p\sigma$  that encompasses the fraction  $p$  of the distribution, and thus has a coverage probability or level of confidence  $p$ , can be readily calculated. Some examples are given in Table G.1.

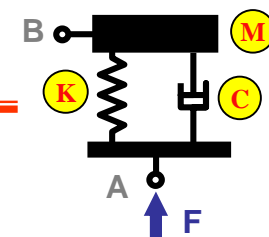
**Table G.1 — Value of the coverage factor  $k_p$  that produces an interval having level of confidence  $p$  assuming a normal distribution**

Level of confidence $p$ (percent)	Coverage factor $k_p$
68,27	1
90	1,645
95	1,960
95,45	2
99	2,576
99,73	3



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International vocabulary of  
metrology – Basic and general  
concepts and associated terms  
(VIM)

3rd edition

2008 version with minor corrections

Vocabulaire international de  
métrologie – Concepts  
fondamentaux et généraux et  
termes associés (VIM)

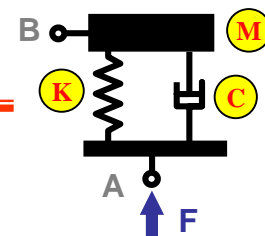
3<sup>e</sup> édition

Version 2008 avec corrections mineures

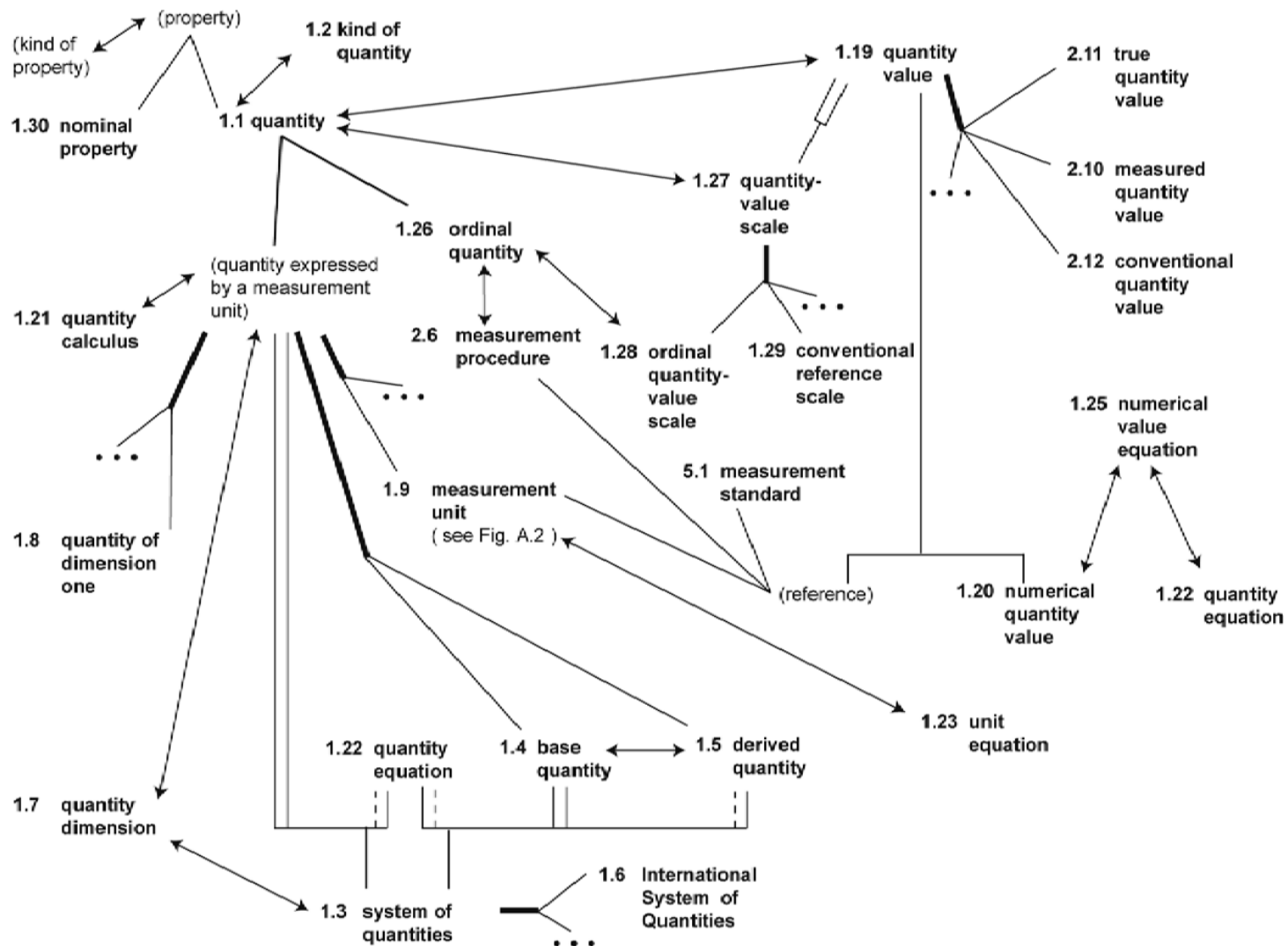


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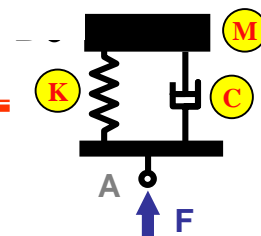


# Quantity

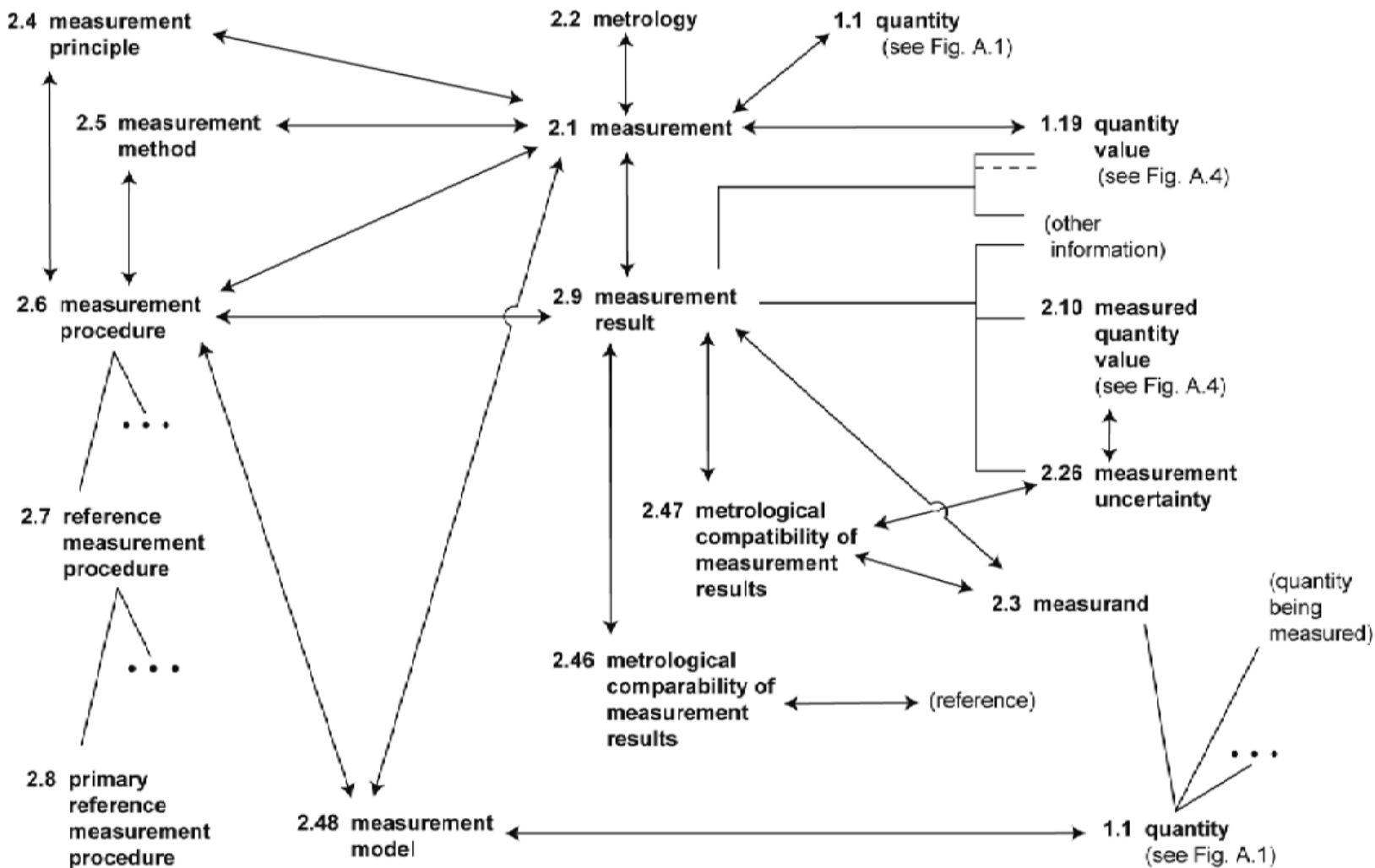


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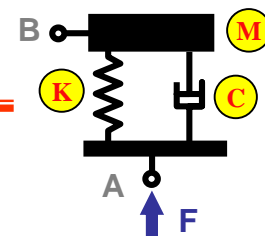


# Measurement



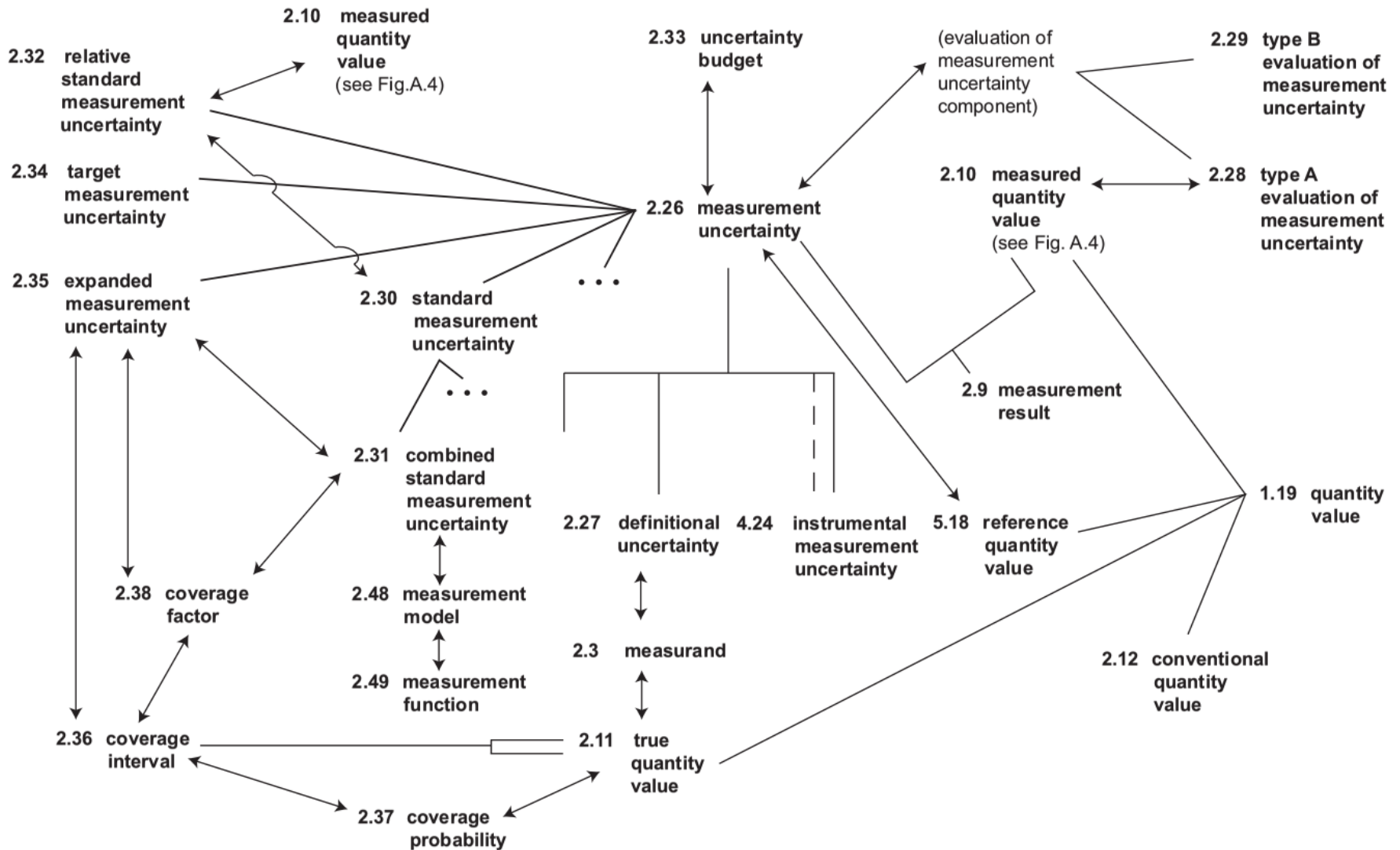
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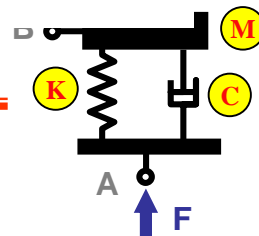


# Uncertainty

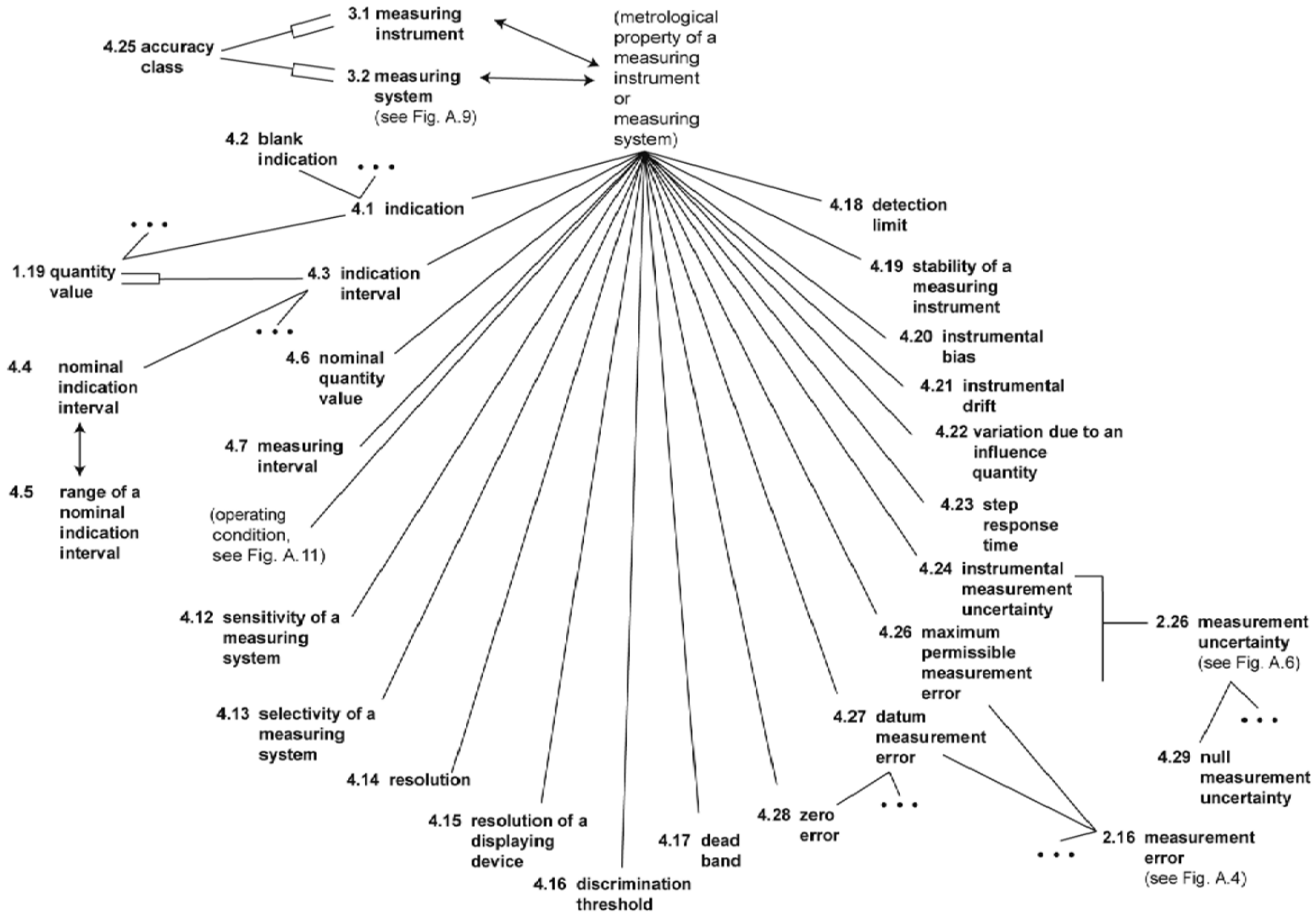


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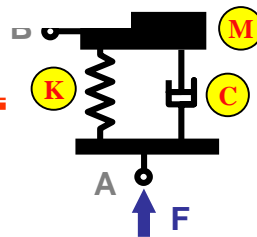


# Metrological properties of a measuring instrument or measuring system



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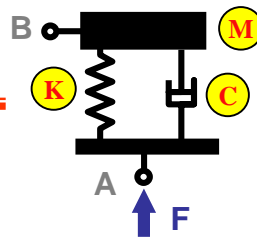


*«Sulla inutilità e nocività della distinzione fra  
approccio bayesiano e frequentista nella scienza  
e tecnica delle misure»*

*Giovanni Battista Rossi*  
Università degli Studi di Genova - Italia



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# Natura della probabilità

Nocciolo del problema:

La probabilità *ha carattere*

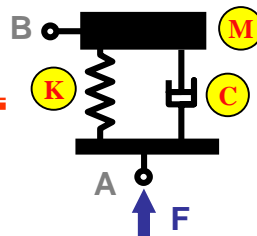
- **Ontico**, o
- **Epistemico**?

**Proprietà ontica:** riguarda le cose in sé, per come *sono realmente*

**Proprietà epistemica:** riguarda il nostro modo di vedere (descrivere, interpretare...) le cose, riguarda la nostra conoscenza delle cose



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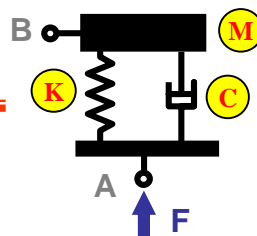
## Ruolo del modello nella scienza e nella ingegneria

**Modello:** un sistema astratto che descrive, da un certo punto di vista ed entro certi limiti, un sistema reale (o una classe di sistemi reali)

*La scienza (e la ingegneria, in quanto scienza applicata) non parla mai direttamente delle cose ma ne parla solo attraverso la mediazione di un modello.*

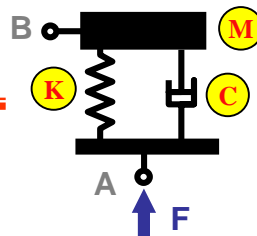
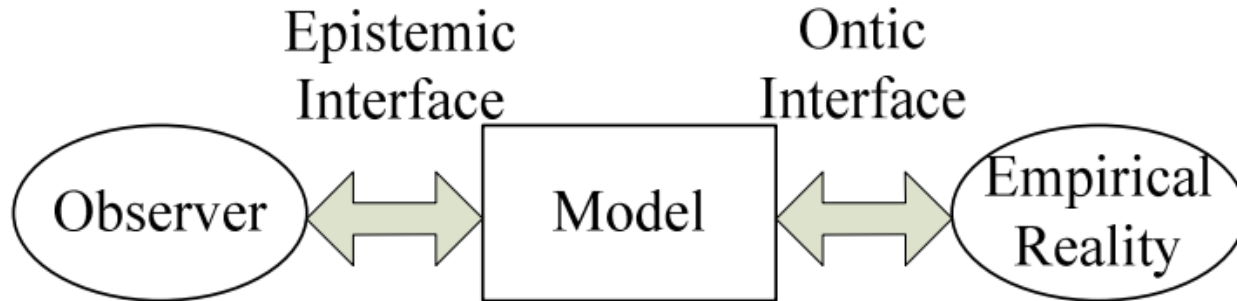


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## Il modello come intermediario fra osservatore e oggetto

- Il modello, in quanto costruito dall'osservatore, è formulato in accordo con le sue categorie mentali (**carattere epistemico**)
- Il modello «coglie qualcosa» della realtà («morde nella realtà», F. Barone) (**carattere ontico**, posizione di realismo moderato)
- Le proprietà introdotte mediante il modello (le sole suscettibili di un discorso scientifico-ingegneristico) non sono né puramente ontiche né puramente epistemiche, ma partecipano, come il modello entro cui sussistono) di entrambi gli aspetti

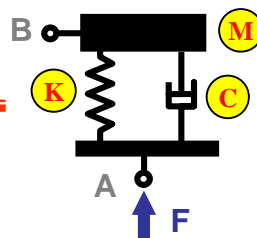


# Modelli deterministici e modelli probabilistici

- *Modello: sistema astratto, cioè insieme di elementi (astratti) fra cui sussistono relazioni*
- **Modello deterministico**: costituito da un sistema le cui relazioni possono essere solo vere o false
- **Modello non deterministico**: almeno una delle relazioni si sottrae ad una logica «vero/falso»
- **Modello probabilistico**: almeno una delle relazioni è di tipo probabilistico



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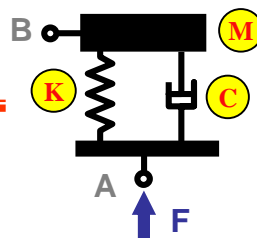


## Che cosa è la probabilità (nelle misure)?

- *Risposta semplice: La probabilità è **uno strumento matematico** (le cui proprietà sono definite in modo rigoroso e assiomatico) che permette di sviluppare modelli probabilistici*
- *Risposta più precisa: La probabilità è **una logica** (una semantica) che interviene nei modelli probabilistici*



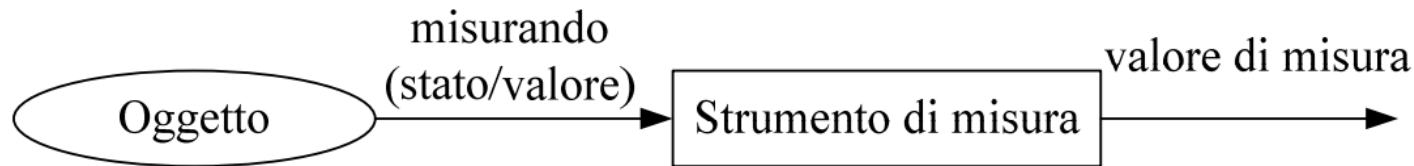
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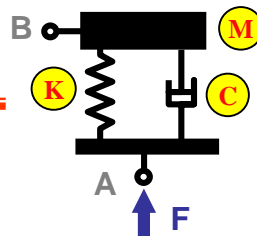


# Modello del processo di misurazione

- Elementi (sottosistemi e proprietà):
  - Un oggetto (in senso lato) portatore della proprietà di nostro interesse
  - La proprietà stessa, manifestata dall'oggetto (il misurando)
  - Lo strumento di misura
  - Il risultato finale (valore di misura)
- Relazioni
  - Una relazione orientata, nel senso causa-effetto, fra tali elementi

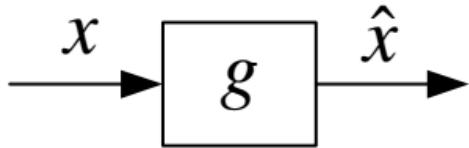


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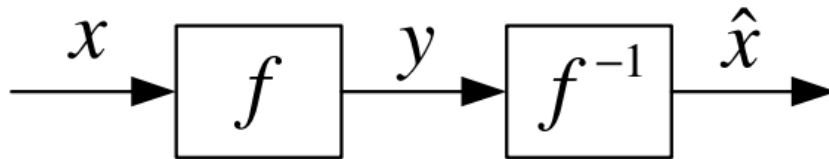


# Modello matematico deterministico

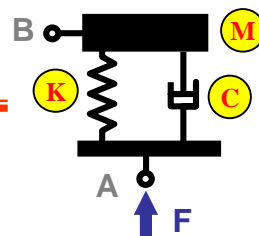
Misura statica di una singola grandezza



Distinzione fra la fase di trasduzione o osservazione e fase di restituzione;  
 $f$  = funzione di taratura

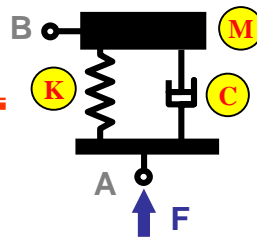
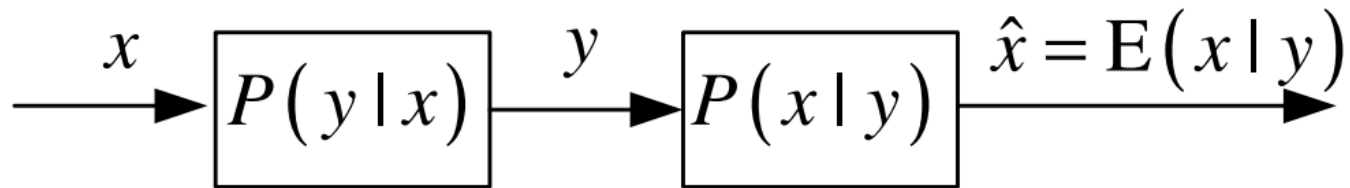


Utile per:  
Progetto di massima  
Confronto fra soluzioni diverse  
...



# Modello probabilistico

- Utile per valutare l'incertezza di misura
- Formalmente: si ottiene dal precedente interpretando le funzioni coinvolte come funzioni probabilistiche
- Interpretazione della probabilità: strumento (logico) matematico che mi permette di trasformare il modello originario in modo da renderlo atto ad esprimere l'incertezza
- NB: l'inversione probabilistica si può ottenere mediante la regola di Bayes-Laplace

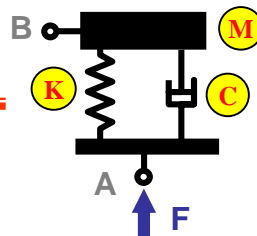


## Sulla natura della probabilità (nelle misure, sintesi)

- Il processo di misurazione può essere studiato, progettato, caratterizzato, utilizzato, da un punto di vista scientifico-ingegneristico, *solo attraverso un modello*, adatto allo scopo.
- Qualora si sia interessati ad evidenziare, esprimere, valutare, *l'incertezza di misura*, una possibile (rimarchevole) opzione è quella di utilizzare *un modello probabilistico*.
- *La probabilità è quello strumento (logico) matematico che ci permette di formulare un tale modello.*



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## Due punti di vista sul processo di misurazione nel suo insieme

A. Descrizione “classica”: il valore di misura è la somma del valore del misurando e dell’errore

$$\hat{x} = x + e$$

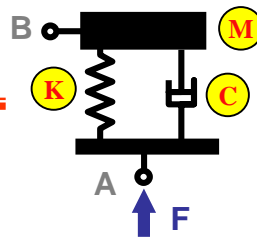
B. Descrizione “GUM-oriented”: il valore del misurando è quel valore che si otterrebbe applicando al valore di misura una opportuna “correzione”

$$x = \hat{x} + c$$

***Come li possiamo interpretare?***



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# Interpretazione mediante la nozione di modello

- **A e B sono due modelli, perfettamente compatibili ( $c = - e$ ), che esprimono *due punti di vista diversi***
- A esprime il punto di vista del costruttore/venditore dello strumento di misura
- B esprime il punto di vista dell'utilizzatore dello strumento di misura

***Quale interpretazione preferite?***



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